An extension problem for fractional powers of generators

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In this talk, we extend results of Caffarelli-Silvestre and Stinga-Torrea regarding a characterization of fractional powers of differential operators via an extension problem. Our main result applies to generators A of α-times integrated semigroups, in particular to purely imaginary operators of Schrödinger type and Laplacians defined on Riemannian manifolds or Lie groups. On the way, we give an integral formula for the fractional operator $(-A)^\sigma$, $0 < \sigma < 1$, that generalizes the well-known formula for generators of continuous semigroups. Moreover, a solution to the extension problem can be expressed in terms of the fractional operator. Previous results on the growth of perturbations of α-times integrated semigroups are also improved, that could be of independent interest.

The talk is based on a joint work with José E. Galé and Pablo Raúl Stinga.