On Titchmarsh-Weyl functions and eigenfunction expansions of first-order symmetric systems

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Assume that H and $\widehat{H}$ are finite dimensional Hilbert spaces, $\mathbb{H} = H \oplus \widehat{H} \oplus H$, $[\mathbb{H}]$ is the set of all bounded operators in $\mathbb{H}$ and $J \in [\mathbb{H}]$ is a signature operator of the form

$$J = \begin{pmatrix} 0 & 0 & -I_H \\ 0 & iI_{\widehat{H}} & 0 \\ I_H & 0 & 0 \end{pmatrix} : H \oplus \widehat{H} \oplus H \to H \oplus \widehat{H} \oplus H.$$

We will discuss first-order symmetric systems

$$Jy'(t) - B(t)y(t) = \lambda \Delta(t) y + \Delta(t) f(t) \tag{1}$$

with the $[\mathbb{H}]$-valued coefficients $B(t) = B^*(t)$ and $\Delta(t) \geq 0$ defined on an interval $\mathcal{I} = [a, b\rangle$ with the regular endpoint a. In the case $\widehat{H} = \{0\}$ system (1) turns into the Hamiltonian system. We assume the deficiency indices $n_\pm$ of the corresponding minimal relation T_{min} to be arbitrary (possibly unequal).

Our approach is based on the concept of a decomposing boundary triplet for the maximal relation $T_{max}(= T_{min}^*)$ introduced in [4] (another construction of a boundary triplet for T_{max} can be found in [1]). This enables us to describe self-adjoint and λ-depending Nevanlinna boundary conditions which are analogs of separated self-adjoint boundary conditions for Hamiltonian systems (1). With a boundary value problem involving such conditions we associate the m-function $m(\cdot) : \mathbb{C} \setminus \mathbb{R} \to [H \oplus \widehat{H}]$. In the case of the Hamiltonian system with equal deficiency indices $n_+ = n_-$ of T_{min} the m-function $m(\cdot)$ coincides with the Titchmarsh-Weyl coefficient. In the simplest case of minimal (unequal) deficiency indices $n_\pm$ the main part of the m-function

$m(\cdot)$ coincides with the rectangular Titchmarsh-Weyl coefficient introduced by Hinton and Schneider in [3].

It turns out that $m(\cdot)$ is a Nevanlinna function and its spectral function $\Sigma(\cdot) : \mathbb{R} \to [H \oplus \widehat{H}])$ is a spectral function of the Fourier transform $L^2_\Delta(\mathcal{I}) \ni f \to g_f \in L^2(\Sigma, H \oplus \widehat{H})$ with the minimally possible dimension $d = \dim(H \oplus \widehat{H})$. We parametrize all m-functions $m(\lambda)$ (and, consequently, all spectral functions $\Sigma(t)$) in terms of the Nevanlinna boundary parameter at the singular endpoint b. Such a parameterization is given by formula similar to the known Krein formula for resolvents.

Application of these results to differential expressions $l[y]$ of an odd order enables us to complete the results by Everitt and Krishna Kumar [2] on the Titchmarsh-Weyl theory of $l[y]$.

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References

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