Spectral gaps of the 1-d Schrödinger operators with periodic distributional potentials

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On the complex Hilbert space $L^2(\mathbb{R})$ we study the 1-dimensional Schrödinger operators $\mathrm{S}(q)$ with 1-periodic real-valued distributional potentials $q(x)$:

$$\mathrm{S}(q)u := -u'' + q(x)u, \qquad q(x) = \sum_{k\in\mathbb{Z}} \widehat{q}(k)e^{i\,2k\pi x} \in H^{-1}(\mathbb{T}).$$

The operators $\mathrm{S}(q)$ are well defined on the Hilbert space $L^2(\mathbb{R})$ in the following equivalent different ways: as form-sums, as quasi-differentials ones, as a limit in norm resolvent sense of the sequence of operators with smooth potentials. The operators $\mathrm{S}(q)$ are self-adjoint and lower semi-bounded. Their spectra are absolutely continuous and have a band and gap structure, see [1] and references therein.

In the talk we discuss the behaviour of the lengths of spectral gaps

$$\gamma_{\mathbf{q}} := \{\gamma_{\mathbf{q}}(n)\}_{n\in\mathbb{N}}$$

of the operators $\mathrm{S}(q)$ in terms of the behaviour of the Fourier coefficients $\{\widehat{q}(k)\}_{k\in\mathbb{Z}}$ of the potentials q with respect to appropriate weight spaces, that is by means of potential regularity [2,3]. We find necessary and sufficient conditions the sequence $\gamma_{\mathbf{q}}$ to be *convergent to zero*, to be *bounded/unbounded.*

The talk is based on a joint work with V. Mikhailets.

References

[1] V. Mikhailets, V. Molyboga, *One-dimensional Schrödinger operators with singular periodic potentials*, MFAT **14** (2008), no. 2, 184–200.

[2] V. Mikhailets, V. Molyboga, *Hill's potentials in Hörmander spaces and their spectral gaps*, MFAT **17** (2011), no. 3, 235–243.

[3] V. Mikhailets, V. Molyboga, *Smoothness of Hill's potential and lengths of spectral gaps*, OT: Adv. & Appl., Vol. 221 (2012), 467–478.