The a priori $\tan\Theta$ theorem for spectral subspaces

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Let A be a self-adjoint operator on a separable Hilbert space $\mathfrak{H}$. Assume that the spectrum of A consists of two disjoint components σ_0 and σ_1 such that the set σ_0 lies in a finite gap of the set σ_1. Let V be a bounded self-adjoint operator on $\mathfrak{H}$ off-diagonal with respect to the partition $\mathrm{spec}(A) = \sigma_0 \cup \sigma_1$. It is known that if $\|V\| < \sqrt{2}d$, where $d = \mathrm{dist}(\sigma_0, \sigma_1)$, then the perturbation V does not close the gaps between σ_0 and σ_1 and the spectrum of the perturbed operator $L = A+V$ consists of two isolated components ω_0 and ω_1 originating from σ_0 and σ_1, respectively. We prove that the bound $\|V\| < \sqrt{2}d$ also implies the following (sharp) norm estimate:

$$\|\mathsf{E}_A(\sigma_0) - \mathsf{E}_L(\omega_0)\| \le \sin\left(\arctan\frac{\|V\|}{d}\right),$$

where $\mathsf{E}_A(\sigma_0)$ and $\mathsf{E}_L(\omega_0)$ are the spectral projections of A and L associated with the spectral sets σ_0 and ω_0, respectively.

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