Eigenvalues in gaps of selfadjoint operators in Pontryagin spaces

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Given two selfadjoint operators A and B in a Hilbert space with n-dimensional resolvent difference, i.e.

$$\dim \operatorname{ran} \left((A-i)^{-1} - (B-i)^{-1} \right) = n,$$

and a gap Δ in the essential spectrum of A (and thus also of B), it is well known that

$$|\operatorname{eig}(A,\Delta) - \operatorname{eig}(B,\Delta)| \le n,$$

where $\operatorname{eig}(\cdot,\Delta)$ denotes the number of eigenvalues (counting multiplicities) of the respective operator in Δ. In this talk we present a result which generalizes this theorem to Pontryagin spaces. To be precise, if A and B are selfadjoint operators in a Pontryagin space $(\Pi_\kappa, [\cdot,\cdot])$ with n-dimensional resolvent difference and Δ is gap in the essential spectra of A and B, we prove that

$$|\operatorname{sig}(\mathcal{L}_\Delta(A)) - \operatorname{sig}(\mathcal{L}_\Delta(B))| \le n,$$

where $\operatorname{sig}(\mathcal{M})$ denotes the signature difference of the inner product $[\cdot,\cdot]$ on the subspace $\mathcal{M}$, and $\mathcal{L}_\Delta(\cdot)$ is the spectral subspace of the respective operator corresponding to Δ. In particular, this implies that

$$|\operatorname{eig}(A,\Delta) - \operatorname{eig}(B,\Delta)| \le n + 2\kappa.$$

The latter estimate is sharp, as will be shown by means of simple examples.

We apply the general result to a class of eigenvalue depending boundary value problems for Sturm-Liouville operators.

The talk is based on joint work with J. Behrndt.