# Inverse spectral problems for Dirac operators with summable matrix-valued potentials

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We solve the direct and inverse spectral problems for self-adjoint Dirac operators $T_q$ generated by the differential expressions

$$\mathfrak{t}_q := \frac{1}{\mathrm{i}} \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \frac{\mathrm{d}}{\mathrm{d}x} + \begin{pmatrix} 0 & q \\ q^* & 0 \end{pmatrix}$$

and the boundary conditions $y_1(0) = y_2(0)$, $y_1(1) = y_2(1)$. Here $q$ is an $r \times r$ matrix-valued function with entries belonging to $L_p(0,1)$, $p \in [1, \infty)$, and $I$ is the identity $r \times r$ matrix.

Namely, the spectrum of the operator $T_q$ consists of countably many isolated real eigenvalues of finite multiplicity, accumulating only at $+\infty$ and $-\infty$. We denote by $\lambda_j(q)$, $j \in \mathbb{Z}$, the pairwise distinct eigenvalues of the operator $T_q$ labeled in increasing order so that $\lambda_0(q) \le 0 < \lambda_1(q)$. Further, let $m_q$ stand for the Weyl-Titchmarsh function of the operator $T_q$. The function $m_q$ is an $r \times r$ matrix-valued meromorphic Herglotz function and $\{\lambda_j(q)\}_{j \in \mathbb{Z}}$ is the set of its poles. We set

$$\alpha_j(q) := - \operatorname*{res}_{\lambda = \lambda_j(q)} m_q(\lambda), \qquad j \in \mathbb{Z},$$

and call $\alpha_j(q)$ the *norming matrix* of the operator $T_q$ corresponding to the eigenvalue $\lambda_j(q)$.

The sequence $\mathfrak{a}_q := ((\lambda_j(q), \alpha_j(q)))_{j \in \mathbb{Z}}$ will be called the *spectral data* of the operator $T_q$, and the matrix-valued measure

$$\mu_q := \sum_{j=-\infty}^{\infty} \alpha_j(q) \delta_{\lambda_j(q)},$$

where $\delta_\lambda$ is the Dirac delta-measure centered at the point $\lambda$, will be called its *spectral measure*. We give a complete description of the class of the spectral data for the operators under consideration (which is equivalent to description of the class of the spectral measures), show that the spectral data determine the operator uniquely and suggest an efficient method for reconstructing the operator from the spectral data.

The talk is based on a joint work with Ya. Mykytyuk.