Existence of maximal semidefinite invariant subspaces and semigroup properties of some classes of ordinary differential operators

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We examine differential operators of the form

$$Lu = \frac{1}{g(x)} L_0 u, \quad x \in (a, b), \tag{1}$$

where L_0 is an ordinary differential operator of order $2m$ defined by the differential expression

$$L_0 u = \sum_{i,j=0}^{m} \frac{d^i}{dx^i} a_{ik}(x) \frac{d^j u}{dx^j} \quad (x \in (a, b)) \tag{2}$$

and the boundary conditions

$$B_k u = \sum_{i=0}^{2m-1} (\alpha_{ik} u^{(i)}(a) + \beta_{ik} u^{(i)}(b)) = 0 \ (k = 1, 2, \ldots, 2m), \tag{3}$$

where we should cancel the corresponding summands if $b = +\infty$ or $a = -\infty$. It is possible that $(a, b) = \mathbb{R}$. The real-valued function $g(x)$ in (1) changes its sign on (a, b). We assume that the operator L is J-dissipative in the Krein space $F_0 = L_{2,g}(a, b)$, where an inner product and an indefinite inner product are defined by the equalities

$$(u, v)_0 = \int_a^b |g(x)| u(x) \overline{v(x)}\, dx, \quad [u, v]_0 = \int_a^b g(x) u(x) \overline{v(x)}\, dx.$$

It is possible that $0 \in \sigma(L)$, i. e., we consider the singular case too. Under certain additional conditions on the behavior of the function $g(x)$, we prove that there exist maximal semidefinite invariant subspaces $H^{\pm}$ of L such that $F_0 = H^+ + H^-$ (the sum is direct) and the restrictions $\pm L|_{H^{\pm}}$ are the generators of analytic semigroups.