The finite section method for infinite Vandermonde matrices

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The finite section method for infinite Vandermonde matrices is the focus of this talk. More precisely, we shall consider an infinite system of equations of the form Ax = d, where

$$A = \begin{pmatrix} 1 & 1 & 1 & \dots \\ a_0 & a_1 & a_2 & \dots \\ a_0^2 & a_1^2 & a_2^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \qquad \qquad d = \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \end{pmatrix}$$

Assuming that all a_j 's are different, the $n \times n$ finite sections A_n of the matrix A will all be invertible. Hence for each n there is a unique solution of $A_n x_n = P_n d$, where P_n is the projection onto the first n coordinates. Embedding x_n into an infinite vector by continuing with zeros, we can study the question whether in some topology these vectors converge to a solution of Ax = d.

It will be shown that for a large class of infinite Vandermonde matrices the finite section method indeed converges in l_1 sense if the right hand side d of the equation is in a suitably weighted $l_1(\alpha)$ space. More explicit results are obtained for a wide class of examples.

This is joint work with András Serény.