Spectral portraits of non-self-adjoint Sturm-Liouville operators with small physical parameter

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We deal with spectral problems of the form $-i\varepsilon y'' + q(x)y = \lambda y$ where λ and ε are the spectral and physical parameters, respectively. The potential $q(x)$ is assumed to be analytic with some additional properties. The case $q(x) = -q(-x)$, $x \in (a, a) \subseteq \mathbb{R}$ corresponds to the so-called PT-symmetric Sturm-Liouville operators. The aim is to understand the spectrum behavior as $\varepsilon \to 0$. It turns out that the spectrum concentrates along some critical curves in the complex plane which form "the limit spectral graph". The problem is to understand the geometry of this graph, to find analytic formulae describing its parts and to write the asymptotic eigenvalue distribution (uniformly, as $\varepsilon \to 0$ along the limit spectral curves. All these problems will be discussed as well as applications in hydrodynamics.