Extinction of solution of semilinear parabolic equations with degenerate absorption potential

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The poster is devoted to the behavior of energy (generalized) solutions for a wide class of semilinear parabolic equations. We investigate a model Cauchy-Neumann problem for parabolic equations of non-stationary diffusion-semi-linear absorption with a degenerate absorption potential. More precisely, the following problem is considered:

$$\big(|u|^{q-1}u\big)_t - \sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(|\nabla_x u|^{q-1}\frac{\partial u}{\partial x_i}\right) + a_0(x)|u|^{\lambda-1}u = 0 \text{ in } \Omega\times(0,T), \quad (1)$$

$$\frac{\partial u}{\partial n}\Big|_{\partial\Omega\times[0,T]} = 0, \quad (2)$$

$$u(x,0) = u_0(x), \quad x\in\Omega. \quad (3)$$

Here $q > 1$, $0 < \lambda < 1$, and $a_0(x) \geq 0$ is an arbitrary continuous function. The initial function $u_0(x)$ is from $L_2(\Omega)$, where $\Omega \subset \mathbb{R}^N (N \geqslant 1)$ is a bounded domain with C^1 - boundary. The origin belongs to Ω ($0 \in \Omega$).

The main focus of our study is the long-time extinction property for solutions to the initial-boundary problem (1), (2), (3). We obtain a sharp condition on the degeneration of the potential $a_0(x)$ that guarantees the long-time extinction.

Let $a_0(x)$ be a potential satisfying the inequality

$$a_0(x) \geq c_0 \exp\Big(-\frac{\omega(|x|)}{|x|^{q+1}}\Big), \quad x\in\Omega\setminus\{0\},$$

where $c_0 > 0$ is a constant, and $\omega(\cdot)$ is an arbitrary function such that

(A) $\omega(\tau) > 0$ $\forall\tau > 0$, (B) $\omega(0) = 0$, (C) $\omega(\tau) \to 0$ as $\tau \to 0$ monotone.

Theorem. *Let* $u_0(x) \in L_2(\Omega)$*. Let* $\omega(\cdot)$ *be a continuous nondecreasing function that satisfies assumptions* (A), (B), (C) *and the following main condition:*

$$\int_0^c \frac{\omega(\tau)}{\tau} d\tau < \infty.$$

Suppose also that $\omega(\cdot)$ *satisfies the technical condition*

$$\frac{\tau\,\omega'(\tau)}{\omega(\tau)} \le 1-\delta \qquad \forall\ \tau \in (0,\tau_0), \quad \tau_0>0, \quad 0<\delta<1.$$

Then an arbitrary energy solution $u(x,t)$ *of problem* (1), (2), (3) *vanishes on* Ω *in a finite time* $T<\infty$*.*

References

[1] J. Diaz, L. Veron, Local vanishing properties of solutions of elliptic and parabolic quasilinear equations, *Trans. Amer. Math. Soc.* **290:2** (1985), p. 787–814.

[2] S. Antontsev, J. Diaz, S. I. Shmarev, The Support Shrinking Properties for Solutions of Quasilinear Parabolic Equations with Strong Absorption Terms, *Annales de la Faculte des Sciences de Toulouse Math.* **6:4** (1995), p. 5–30.

[3] Y. Belaud, A. Shishkov, Long-time extinction of solutions of some semilinear parabolic equations, *Journal of Differential Equations* **238** (2007), p. 64–86.