Convergence analysis of high-order time-splitting pseudo-spectral methods for a class of nonlinear Schrödinger equations

Mechthild Thalhammer

In this talk, the issue of favourable numerical methods for the space and time discretisation of low-dimensional nonlinear Schrödinger equations is addressed. The objective is to provide a stability and error analysis of high-accuracy discretisations that rely on spectral and splitting methods. As a model problem, the time-dependent Gross–Pitaevskii equation arising in the description of Bose–Einstein condensates is considered. For the space discretisation pseudo-spectral methods collocated at the associated quadrature nodes are analysed. For the time integration high-order exponential operator splitting methods are studied, where the decomposition of the function defining the partial differential equation is chosen in accordance with the underlying spectral method. The convergence analysis relies on a general framework of abstract nonlinear evolution equations and fractional power spaces defined by the principal linear part. Essential tools in the derivation of a temporal global error estimate are further the formal calculus of Lie-derivatives and bounds for iterated Lie-commutators. Numerical examples for higher-order time-splitting pseudo-spectral methods applied to time-dependent Gross–Pitaevskii equations illustrate the theoretical result.