Extremal sectorial operators and relations

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A (linear) relation S in a Hilbert space H is said to be *sectorial* with vertex at the origin and semiangle α, $\alpha \in [0, \pi/2)$, if

$$|\Im(\varphi', \varphi)| \leq (\tan \alpha)\Re(\varphi', \varphi), \quad \{\varphi, \varphi'\} \in S.$$

A sectorial relation S in a Hilbert space H is said to be *maximal sectorial* if the existence of a sectorial relation T in H with $S \subset T$ implies $S = T$. Sectorial operators or relations have maximal sectorial extensions. In this case S decomposes into the orthogonal sum of a densely defined maximal sectorial operator and a purely multivalued part. A maximal sectorial extension H of S is said to be *extremal* if

$$\inf\{\, \Re(h' - \varphi', h - \varphi) : \{\varphi, \varphi'\} \in S \,\} = 0 \quad \text{for all} \quad \{h, h'\} \in H.$$

The Friedrichs extension and the Kreĭn-von Neumann extension are extremal in this sense. It is shown that all extremal extensions of a sectorial relation can be characterized in terms of factorizations. As in the case of nonnegative relations, the factorizations of the Kreĭn-von Neumann and Friedrichs extensions lead to a novel approach to the transversality and equality of the extreme extensions, and to the notion of positive closability, meaning that the Kreĭn-von Neumann extension is an operator.

The talk is based on a joint work with S. Hassi, A. Sandovici and H. de Snoo.