Hamiltonians and Riccati equations for unbounded control and observation operators

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We consider the control algebraic Riccati equation

$$A^*X + XA - XBB^*X + C^*C = 0$$

for the case that A is a normal operator with compact resolvent, $B \in L(U, H_{-s})$ and $C \in L(H_s, Y)$, $0 \le s \le 1$. Here $H_s \subset H \subset H_{-s}$ are the usual fractional domain spaces corresponding to A. This setting includes the case where B and C correspond to point or boundary control and observation, respectively. We show the existence of infinitely many solutions X of the Riccati equation using invariant subspaces of the Hamiltonian operator matrix

$$T = \begin{pmatrix} A & -BB^* \\ -C^*C & -A^* \end{pmatrix}.$$

Just like in the finite-dimensional case, each such solution corresponds to a certain choice of eigenvalues of T. We also obtain conditions for bounded, nonnegative, and nonpositive solutions. Our main tools are Riesz bases of eigenvectors of T and indefinite inner products.

The talk is based on joint work with B. Jacob and H. J. Zwart.