Spectral Theory and Differential Operators

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BOOK OF ABSTRACTS

Essential spectra of some matrix operators by means measure of noncompactness

Boulbeba Abdelmoumen

Measures of noncompactness have been successfully applied in topology, functional analysis and operator theory. In this paper, we work with the notion of the measures of noncompactness in order to investigate the essential spectra of some matrix operators on Banach spaces. These results are exploited to describe the essential spectra of two-group transport operators.

Local scattering theory and scattering matrices for quantum chains

Vadym Adamyan

The Hamiltonian for carriers (electrons, holes) in a quantum chains constructed from quantum dots and attached straight quantum wires is usually modeled by one-particle Schrödinger operators (Laplace operator + some potentials) or Dirac operators on intricate domains of the three- or two-dimensional space. In fact, scattering processes in real chains are observed only on a rather narrow spectral interval Λ of essential spectrum centered at the Fermi-level. Therefore more simple Hamiltonians in the form of ordinary differrential operators on graphs may exhibit just the same scattering picture on Λ, as the original Hamiltonian of the chain. A key issue for engineering of such a fitting solvable models can be a local scattering theory for a finite spectral interval Λ, especially the local scattering theory for self-adjoint extensions of the same densely defined symmetric operators.

In the talk reported on we review this version of local scattering theory complemented with some new results ("chain rules" for local wave and scattering operators) and develop on this base a perturbation procedure permitting to improve results obtained as a first order approximation in the framework of described above solvable ODO model for real quantum chains with PDO Hamiltonians.

The talk is based on joint works with B. Pavlov.

Relative oscillation theory for Jacobi operators

Kerstin Ammann

Classical oscillation theory for Jacobi operators connects the number of eigenvalues below a given value with the number of sign flips of certain solutions of the underlying difference equation. Considered here will be the difference between the number of eigenvalues of two Jacobi operators which we will connect with the number of sign flips of the Wronskian of two solutions of the underlying difference equations.

The talk is based on a joint work with G. Teschl.

Does diffusion determine the domain?

Wolfgang Arendt

In the talk we will give a survey on answers to Kac's famous question: Can one hear the shape of a drum? The main part of the talk is devoted to a slightly different inverse problem, though. Given two domains, to say that the corresponding Laplace operators with Dirichlet boundary conditions have the same spectrum means that there exists a unitary operator intertwining the corresponding heat semigroups. Kac's originial question is whether this implies that the two underlying domains are congruent. Instead of an intertwining unitary operator we consider an intertwining order isomorphism (i.e. a bijective linear mapping U between the L^2 spaces such that $Uf \leq 0$ iff $f \leq 0$). This may be interpreted by saying that U maps the positive solutions of the diffusion equation to positive solutions. Formulated in this way, the answer is positive: the domains are indeed congruent if such intertwining order isomorphism exists. One may interprete the result by saying that "diffusion determines the domain". But much more is true: also the boundary conditions are determined. In fact, we consider such an order isomorphism which intertwines the Laplacian with a priori different boundary conditions on both sides. The result is that they have to be the same and the domains have to be congruent. Diverse quite subtle regularity considerations are needed even though the results are valid under very mild regularity conditions on the domains. The result also holds on Riemannian Manifolds (joint work with Markus Biegert and Tom ter Elst). In the general case the related regularitiy problems are delicate. However, if both manifolds are compact so that no boundary conditions are imposed, a direct easy proof is possible (see the joint paper with Tom ter Elst below).

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Estimate of the decay exponent of an operator semigroup associated with a second-order linear differential equation

Nikita Artamonov

In Hilbert space H we consider a second-order linear differential equation

$$u''(t) + Du'(t) + Au(t) = 0$$

and related quadric pensil $L(\lambda) = \lambda^2 I + \lambda D + A$ with self-adjoint positive definite operator A. By H_s denote a collection of Hilbert spaces generated by operator $A^{1/2}$, $\|\cdot\|_s$ is a norm on H_s. We will assume that D is a bounded operator acting from H_1 to H_{-1} and

$$\inf_{x \in H_1, x \neq 0} \frac{\mathrm{Re}(Dx, x)_{-1,1}}{\|x\|_1^2} = \delta > 0.$$

(here $(\cdot,\cdot)_{-1,1}$ is a duality pairing on $H_{-1} \times H_1$). The second-order differrntial equation can be linearized as a system $w'(t) = \mathcal{T}w(t)$ in "energy" space $H \times H_1$, where

$$w(t) = \begin{pmatrix} u'(t) \\ u(t) \end{pmatrix} \quad \mathcal{T} = \begin{pmatrix} -D & -A \\ I & 0 \end{pmatrix}.$$

We estimate an exponential decay rate for a semigroup generated by operator $\mathcal{T}$ in the space $H \times H_1$. We also obtained localization of the spectrum of the pencil $L(\lambda)$:

$$\sigma(L) = \sigma(\mathcal{T}) \subset \{\lambda \in \mathbb{C} \mid \mathrm{Re}\lambda \leq -\omega, |\mathrm{Im}\lambda| \leq \kappa(b)|\mathrm{Re}\lambda| + b\}$$

for some positive ω and for all $b > 0$. The work is supported by the Russian Fund for Basic Research (grant No. 11-01-00790)

Product formulas for operator matrix semigroups

András Bátkai

Many systems in physics, biology or engineering can be described by an abstract Cauchy problem on a product Banach space. Unfortunately, unbounded operators on product spaces are in general difficoult to represent as "matrix" operators.

It is also a big problem in applications that complicated systems can be solved directly usually at high cost. The idea of operator splitting is to split the problem into simple sub-problems and then use a product formula to represent the solutions.

In this talk, we present easy to verify conditions implying stability estimates for operator matrix splittings which ensure convergence of the associated Trotter, Strang and weighted product formulas. The results are applied to inhomogeneous abstract Cauchy problems and to boundary feedback systems.

Joint work with Petra Csomós, Klaus-Jochen Engel and Bálint Farkas.

On sectorial classes of inverse Stieltjes functions

Sergey Belyi

We introduce sectorial classes of inverse Stieltjes functions acting on finite-dimensional Hilbert space as well as scalar classes of inverse Stieltjes functions characterized by their limit values. It is shown that a function from these classes can be realized as the impedance function of an L-system whose associated operator $\tilde{\mathbb{A}}$ is sectorial. Moreover, it is established that the knowledge of the limit values of the scalar impedance function allows us to find an angle of sectoriality of operator $\tilde{\mathbb{A}}$ as well as the exact angle of sectoriality of the accretive main operator T of such a system. The corresponding new formulas connecting the limit values of the impedance function and the angle of sectoriality of $\tilde{\mathbb{A}}$ are provided. These results are illustrated by examples of the realizing L-systems based upon the Schrödinger operator on half-line.

The talk is based on a recent joint work with E. Tsekanovskiĭ.

Scattering and inverse scattering for a left-definite Sturm-Liouville problem

Malcolm Brown

This talk reports on work develops a scattering and an inverse scattering theory for the Sturm-Liouville equation $-u'' + qu = \lambda wu$ where w may change sign but $q \geq 0$. Thus the left-hand-side of the equation gives rise to a positive quadratic form and one is led to a left-definite spectral problem. The crucial ingredient of the approach is a generalized transform built on the Jost solutions of the problem and hence termed the Jost transform and the associated Paley-Wiener theorem linking growth properties of transforms with support properties of functions.

One motivation for this investigation comes from the Camassa-Holm equation for which the solution of the Cauchy problem can be achieved by the inverse scattering transform for $-u'' + \frac{1}{4}u = \lambda wu$.

It is based on joint work with Christer Bennewitz (Lund), and Rudi Weikard (Birmingham AL)

On an analogue of formulae Sohotski–Plemelj

Evgen Cheremnikh

Let $H = L^2(0,\infty)$ and $T = S + A^*B$ where $(S\varphi)(\tau) \equiv \tau\varphi(\tau)$, $A, B\colon H \to G$ are integrals operators and G is auxiliary Hilbert space. Let

$$D(\widetilde{S}) = \left\{ \varphi \in H \colon \exists c \equiv c(\varphi)\colon \int\limits_0^\infty |c(\varphi) + s\varphi(s)|^2 \rho(s) ds < \infty \right\}$$

and $\widetilde{S}\varphi(s) = c(\varphi) + s\varphi(s)$, $\varphi \in D(\widetilde{S})$.

Let $\widetilde{T} = \widetilde{S} + A^*B$, $D(\widetilde{T}) = D(\widetilde{S})$ and $T_\xi = (T-\xi)^{-1}$, $\widetilde{T}_\xi = (\widetilde{T}-\xi)^{-1}$. We discuss the relation

$$(T_\sigma\varphi, \psi)_\pm = (\varphi, b_\sigma)(a_\sigma^\pm\psi) + (\widetilde{T}_\sigma\varphi, \psi)_H$$

between the limit values $(T_\sigma\varphi, \psi)_\pm = \lim\limits_{\tau\to+0}(T_{\sigma\pm i\tau}\varphi, \psi)$ and bilinear form of the resolvent $(\widetilde{T}_\sigma\varphi, \psi)_H$ of maximal operator $\widetilde{T}$.

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Asymptotic generalized value distribution of solutions of the Schrödinger equation

Yiannis Christodoulides

We develop the theory of generalized value distribution for the class of functions defined as boundary values of Herglotz functions. One of the main results is an estimate of asymptotic generalized value distribution for Herglotz functions translated by an increment $i\delta$ off the real axis. We also present precise relations between the employed measures, and the link with compositions of Herglotz functions. A case of particular interest which we study is that of the Weyl-Titchmarsh m-function associated with Sturm-Liouville differential operators. In this context, this generalized theory allows for the possibility to describe the asymptotic value distribution of solutions of the Schrödinger equation in terms of other measures than Lebesgue measure.

The inverse spectral transform for the dispersionless Camassa–Holm equation

Jonathan Eckhardt

The dispersionless Camassa–Holm equation

$$\omega_t + 2\omega u_x + \omega_x u = 0, \quad u - u_{xx} = \omega, \qquad x,\, t \in \mathbb{R}$$

can be solved by the inverse spectral method: Given a solution of this equation, some particular (time dependent) spectral quantities of the weighted Sturm–Liouville spectral problems (parametrized by time $t \in \mathbb{R}$)

$$-f''(x) + \frac{1}{4} f(x) = z f(x) \omega(x,t), \qquad x \in \mathbb{R},\ z \in \mathbb{C}$$

evolve according to a simple linear flow, which can be solved explicitly. The transformation which takes some ω to the spectral quantities of the corresponding spectral problem (and hence the Camassa–Holm flow to a simple linear flow) is referred to as the inverse spectral transform. We discuss some properties of this transformation and employ them to derive several facts for the dispersionless Camassa–Holm equation.

The talk is based on joint work with G. Teschl.

The Dirichlet-to-Neumann operator on rough domains

Tom ter Elst

We consider a bounded connected open set $\Omega \subset \mathrm{R}^d$ whose boundary Γ has a finite $(d-1)$-dimensional Hausdorff measure. Then we define the Dirichlet-to-Neumann operator D_0 on $L_2(\Gamma)$ by form methods. The operator $-D_0$ is self-adjoint and generates a contractive C_0-semigroup $S = (S_t)_{t>0}$ on $L_2(\Gamma)$. We show that the asymptotic behaviour of S_t as $t \to \infty$ is related to properties of the trace of functions in $H^1(\Omega)$ which Ω may or may not have. We also show that they are related to the essential spectrum of the Dirichlet-to-Neumann operator.

The talk is based on a joint work with W. Arendt (Ulm).

Spectral estimates for Schrödinger operators with unusual semiclassical behaviour

Pavel Exner

In this talk I am going to discuss spectral estimates for several classes of Schrödinger operators which exhibit a discrete spectrum although the corresponding phase space volume is infinite. The first concerns modifies the well-known example of narrowing potential channels with the aim to show that the spectrum may be purely discrete even for potentials unbounded from below. Next I will consider Dirichlet Laplacians in cusp-shaped regions and derive inequalities of Lieb-Thirring type showing how they depend on the geometry of the regions. This is a common work with Diana Barseghyan.

On the buckling operator of an unbounded operator

Karl-Heinz Förster

Let S be a densely defined, closed and linear operator in a Hilbert space $\mathcal{H}$, let $\mathcal{G}_S$ denote its graph equipped with the graph norm and let i_S denote the canonical injection $\mathcal{G}_S \to \mathcal{H}$.

The buckling operator of S is given as

$$S_B = i_S^* S i_S : \ \mathcal{G}_S \to \mathcal{G}_S.$$

It follows easily that S_B is bounded and has the same numerical range as S; therefore S is symmetric if and only if S_B is selfadjoint. [M. S. Ashbaugh, F. Gesztesy, M. Mitrea, R. Shterenberg and G. Teschl: The Krein-von Neumann Extension and its connection to an abstract buckling problem. Math. Nachr. 283, No. 2, 165-179 (2010)] proved : If S is positive definite then S_B is (for an approbriate scalar product on $\mathcal{G}_S$) unitarily equivalent to the inverse of the reduced Krein-von Neumann extension of S.
We extend this result to the von Neumann extension S_N where

$$\operatorname{Dom}(S_N) = \operatorname{Dom}(S) + \operatorname{Ker}(S^*) \qquad S_N(u+v) = Su.$$

for a symmetric operator S bounded from below.

This is a joint work with N. Lubrich.

Spectral dimension of V–variable fractals

Uta Freiberg

The concept of V–variable fractals (developed by Barnsley, Hutchinson and Stenflo) allows describing new families of random fractals, which are intermediate between the notions of deterministic and of random fractals including random recursive as well as homogeneous random fractals. The parameter V describes the degree of "variability"of the realizations.
Brownian motion and Laplacian can be constructed from the associated Dirichlet forms. The properties of these objects are modified by the random environment. We obtain the spectral dimension (i.e. the exponent of the leading term of the eigenvalue counting function of the Laplacian) by applying Kesten-Furstenberg techniques.

Positive operators arising from contractions

Gyoergy Geher

Let $\mathcal{H}$ be a complex Hilbert space. If $T \in \mathcal{B}(\mathcal{H})$ is a contraction i.e..: $\|T\| \leq 1$, then the sequence $\{T^{*n}T^n\}_{n=1}^{\infty}$ of positive operators is decreasing, so it has a positive limit in the strong operator topology (SOT):

$$A_T = \lim_{n\to\infty} T^{*n}T^n.$$

We say that A_T *is induced by* T, or A_T is the *asymptotic limit* of T. The subspace

$$\mathcal{N}(A_T) = \mathcal{H}_0(T) := \{x \in \mathcal{H} \colon \lim_{n\to\infty} \|T^n x\| = 0\}$$

is hyperinvariant for T and it is called the *stable subspace* of T. The subspace

$$\mathcal{N}(A_T - I) = \mathcal{H}_1(T) := \{x \in \mathcal{H} \colon \lim_{n\to\infty} \|T^n x\| = \|x\|\}$$

is the largest invariant subspace where T is an isometry.

We will describe those positive operators that arises from a contraction in such a way. Moreover we can ensure uniform convergence, and expect from the case when $0 < \dim \mathcal{H}_1 < \aleph_0$ we can choose a co-stable contraction i.e.: $\mathcal{H}_0(T^*) = 0$. After that we give some sufficient condition for two contraction, having the same asymptotic limit.

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Triplets of closely embedded Hilbert spaces and Dirichlet type spaces on the unit polydisc

Aurelian Gheondea

We obtain a general concept of triplet of Hilbert spaces with closed embeddings instead of continuous ones, first by a model associated to a positive selfadjoint operator H that is one-to-one but may not have a bounded inverse. Existence and uniqueness results for these generalized triplets of Hilbert spaces are obtained. As an illustration of the abstract theory we show how rather general weighted L^2 spaces yield this kind of generalized triplets of Hilbert spaces for which the underlying spaces and operators can be explicitly calculated. Then we show that generalized triplets of Hilbert spaces with closed embeddings can be naturally associated to any pair of Dirichlet type spaces $\mathcal{D}_\alpha(\mathbb{D}^N)$ of holomorphic functions on the unit polydisc $\mathbb{D}^N$ and we explicitly calculate the associated operators in terms of reproducing kernels and radial derivative operators. Finally, a rigging of the Hardy space $H^2(\mathbb{D}^N)$ through a scale of Dirichlet type spaces is also presented.

The talk is based on joint work with P. Cojuhari.

Spectral multiplicity for the one-dimensional Schrödinger operator on the line

Daphne Gilbert

We consider the one-dimensional Schrödinger operator H on the real line in the case where Weyl's limit point case holds at both of the infinite endpoints. In this situation it is well-known that the spectral multiplicity of H may be one or two, and that it is often convenient to decompose H into a direct sum of two half-line operators, $H_{-\infty}$ and H_∞, when investigating the spectrum of H.

We will present necessary and sufficient conditions for H to have simple and/or degenerate spectrum, with particular reference to one or more of the following criteria:

(i) the boundary behaviour on the real axis of the corresponding Titchmarsh-Weyl functions, $m_{-\infty}$ and m_∞, which are associated with the Dirichlet half-line operators, $H_{-\infty}$ and H_∞, on $(-\infty, 0]$ and $[0, \infty)$, respectively,

(ii) the asymptotic behaviour of solutions of the Schrödinger equation at each of the limit point endpoints, in terms of the theory of subordinacy,

(iii) the rank of the spectral density for H.

The stability of the absolutely continuous and essential spectra under various perturbations will also be discussed, with a view to enabling the multiplicity properties of a family of operators to be inferred from complete knowledge of the location and multiplicity of the absolutely continuous and essential spectra associated with a single operator. This can be achieved provided the location and multiplicity of the absolutely continuous and essential spectra of the original operator are invariant under the chosen perturbation.

On the Krein and Friedrichs extensions of a positive Jacobi operator

Nataliia Goloshchapova

We describe extremal so-called Friedrichs and Krein extensions of positive Jacobi operator acting in $l^2(\mathbb{N})$ and formally defined from

$$(\mathrm{J}x)_n = a_n x_{n+1} + b_n x_n + a_{n-1} x_{n-1}, \quad a_n \in \mathbb{R}_+,\, b_n \in \mathbb{R}. \qquad (*)$$

Using the technique of boundary triplets and the corresponding Weyl functions (see [2, 3, 4, 5]), we complete and generalize the results obtained by B.M. Brown and J.S. Christiansen [1].

Namely, we parametrize extremal self-adjoint extensions of the initial positive minimal symmetric operator corresponding to $(*)$ in terms of a certain boundary conditions. In addition, we characterize the Krein extension in the case of matrix entries a_n, b_n.

The talk is based on a joint work with A. Ananyeva.

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Eigenvalue asymptotics for nonsmooth singular Green operators

Gerd Grubb

Singular Green operators G appear typically as boundary correction terms in resolvents for elliptic boundary value problems on a domain $\Omega \subset \mathbb{R}^n$, and more generally they appear in the calculus of pseudodifferential boundary problems. In particular, the boundary term in a Krein resolvent formula is a singular Green operator. It is well-known in smooth cases, that when G is of negative order $-t$ on a bounded domain, its s-numbers (eigenvalues, if G is selfadjoint nonnegative) have the behavior

$$s_j(G) \sim cj^{-t/(n-1)} \text{ for } j \to \infty, \qquad (*)$$

governed by the boundary dimension $n-1$. In some nonsmooth cases, upper estimates $s_j(G) \le Cj^{-t/(n-1)}$ are known.

We show that $(*)$ holds when G is a general selfadjoint nonnegative singular Green operator with symbol merely Hölder continuous in x. We also show $(*)$ with $t = 2$ for the boundary term in the Krein resolvent formula comparing the Dirichlet and a Neumann-type problem for a strongly elliptic second-order differential operator (not necessarily selfadjoint), with coefficients in $W^1_p(\Omega)$ for some $p > n$.

Global solutions of the two-component Camassa–Holm system

Katrin Grunert

The two-component Camassa-Holm (2CH) system

$$\begin{aligned} u_t - u_{txx} + \kappa u_x + 3uu_x - 2u_x u_{xx} - uu_{xxx} + \eta\rho\rho_x = 0, \\ \rho_t + (u\rho)_x = 0, \end{aligned}$$

with arbitrary $\kappa \in \mathbb{R}$ and $\eta \in (0, \infty)$, serves as a model for shallow water. Furthermore, it is a generalization of the famous Camassa–Holm (CH) equation which has been studied intensively. Thus naturally the question arises which results derived for the CH equation are also valid for the 2CH system. In this talk we will show how to describe global solutions. This question is of special interest since the 2CH system, like the CH equation, enjoys wave breaking and in general there are two possibilities how to continue solutions thereafter. Namely, either the energy is preserved which yields conservative solutions or if energy vanishes from the system, we obtain dissipative solutions. Additionally, we will admit initial data and hence solutions with nonvanishing asymptotics.

This talk is based on joint work with H. Holden and X. Raynaud.

Variation of discrete spectra of non-selfadjoint operators

Marcel Hansmann

A classical result of Kato states that for two bounded selfadjoint operators A, B with $B - A \in \mathcal{S}_p$, the Schatten ideal of order p, one has

$$\sum_{\lambda \in \sigma_d(B)} \operatorname{dist}(\lambda, \sigma(A))^p \leq \|B - A\|_{\mathcal{S}_p}^p, \quad p \geq 1,$$

where $\sigma_d(B)$ denotes the discrete spectrum of B.

In this talk, I will present some recent extensions of Kato's result to the case of non-selfadjoint operators.

Some new subclasses of generalized Nevanlinna functions

Seppo Hassi

Given an arbitrary symmetric rational function r we introduce and study the following subclasses of (generalized) Nevanlinna functions $\mathcal{N}_\kappa$ generated by r:

$$\mathcal{N}_\kappa^{\tilde{\kappa}}(r) = \{\, Q \in \mathcal{N}_\kappa : rQ \in \mathcal{N}_{\tilde{\kappa}}, \quad \kappa, \tilde{\kappa} \in \mathbb{N}\,\}.$$

The canonical factorizations for the functions $Q \in \mathcal{N}_\kappa$ and $rQ \in \mathcal{N}_\kappa^{\tilde{\kappa}}(r)$ are also established and the connection between the corresponding operator models (realizations) is explained. Various special cases of these subclasses, known in the earlier literature, are indicated.

The talk is based on a joint work with H.L. Wietsma (Vaasa).

The jump problem for holomorphic functions on non-rectifiable arc and $\overline{\partial}$−equation

Boris Kats

The jump problem is the following well known boundary value problem for analytical function. Let Γ be a given arc on the complex plane $\mathbb{C}$. It is required to find analytical in $\overline{\mathbb{C}} \setminus \Gamma$ function $\Phi(z)$ such that

$$\Phi^+(t) - \Phi^-(t) = f(t), t \in \Gamma, \tag{2}$$

where $\Phi^+(t)$ and $\Phi^-(t)$ are limit values of $\Phi(z)$ at point $t \in \Gamma$ from the left and from the right correspondingly, and $f(t), t \in \Gamma$, is given function. If arc Γ is piecewise–smooth, then a solution of this problem is given by the Cauchy integral

$$\Phi(z) = \frac{1}{2\pi i}\int_\Gamma \frac{f(t)dt}{t-z}.$$

It is convolution of distributions $E(z) \equiv (\pi i z)^{-1}$ and $\int_\Gamma f(t)\cdot dt$, where

$$\left\langle \int_\Gamma f(t)\cdot dt, \phi \right\rangle \equiv \int_\Gamma f(t)\phi(t)dt, \phi \in C_0^\infty(\mathbb{C}).$$

As E is fundamental solution of differential operator $\overline{\partial} \equiv \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$, since the differential equation

$$\overline{\partial}\Phi = \int_\Gamma f(t)\cdot dt \tag{3}$$

is distributional version of the jump problem.

If the arc Γ is not rectifiable, then both the Cauchy integral and the distribution $\int_\Gamma f(t) \cdot dt$ lose their certainty. In the present work we construct generalization of $\int_\Gamma f(t) \cdot dt$ for non-rectifiable arcs such that solution of the equation (3) with this generalization in the right side is also solution of the problem (2) on non-rectifiable arc.

Spectral theory of the truncated Fourier operator

Victor Katsnelson

The truncated Fourier operator $\mathfrak{F}_{\mathbb{R}_+}$ acts in the space $L^2(\mathbb{R}_+)$ and is an operator of the form

$$\mathfrak{F}_{\mathbb{R}_+} = P\mathfrak{F}P,$$

where $\mathfrak{F}$ is the Fourier-Plancherel operator (acting in $L^2(\mathbb{R})$) and P is the natural projector from $L^2(\mathbb{R})$ onto $L^2(\mathbb{R}_+)$.

The spectral theory of the operator $\mathfrak{F}_{\mathbb{R}_+}$ and the appropriate functional calculus are developed.

Volume growth and spectra of Dirichlet forms

Matthias Keller

In Riemannian geometry the exponential volume growth yields an upper bound from the bottom of the essential spectrum. In contrast, there are graphs of polynomial growth that already have positive bottom of the essential spectrum, if the volume is measured with respect to the natural graph distance. This disparity can be resolved in the common framework of regular Dirichlet forms. There, one has a concept of intrinsic metrics. If the volume is measured with such an intrinsic metric, then the classical result can be achieved in great generality. Moreover, for graphs one can relate the natural graph distance to a special intrinsic metric. In this way one finds that the threshold for positive bottom of the essential spectrum lies at cubic growth with respect to the natural graph distance.
This is joint work with Sebastian Haeseler and Radoslaw Wojciechowski.

On the Sz.-Nagy–Foias functional calculus

Laszlo Kerchy

The Sz.-Nagy–Foias fuctional calculus, operating with bounded analytic functions on the unit disc, is an efficient tool in the study of Hilbert space contractions. We examine its role in the characterization of stability and quasianalycity. Mapping theorems are discussed in connection with important spectral invariants.

On Global Attractors of Nonlinear Hyperbolic PDEs

Alexander Komech

We consider Klein-Gordon and Dirac equations coupled to U(1)-invariant nonlinear oscillators. The solitary waves of the coupled nonlinear system form two-dimensional submanifold in the Hilbert phase space of finite energy solutions. Our main results read as follows:

Theorem *Let all the oscillators be strictly nonlinear. Then any finite energy solution converges, in the long time limit, to the solitary manifold in the local energy seminorms.*

The investigation is inspired by Bohr's postulates on transitions to quantum stationary states. The results are obtained for:

- 1D KGE coupled to one oscillator [1, 2, 3], and to finite number of oscillators [4];
- nD KGE and Dirac eqns coupled to one oscillator via mean field interaction [5, 6].

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On asymptotic stability of kinks for relativistic Ginzburg-Landau equation

Elena Kopylova

We consider nonlinear relativistic wave equation in one space dimension

$$\ddot{\psi}(x,t) = \psi''(x,t) + F(\psi(x,t)), \quad x \in \mathbb{R}, \quad F(\psi) = -U'(\psi), \tag{4}$$

where $U(\psi)$ is a potential of Ginzburg-Landau type

$$U(\psi) \sim (\psi^2 - 1)^2/4.$$

The kink is a nonconstant finite energy solution of stationary equation

$$s(x) \sim \tanh x/\sqrt{2}.$$

The corresponding moving kinks or solitary waves

$$s_{q,v}(t) = s(x - vt - q), \quad q, v \in \mathbb{R}, \quad |v| < 1, \quad \gamma = 1/\sqrt{1 - v^2}$$

are the solutions to equation (1). Our main results are the following asymptotics

$$(\psi(x,t), \dot{\psi}(x,t)) \sim (s_{q_\pm, v_\pm}(x - v_\pm t - q_\pm), \dot{s}_{q_\pm, v_\pm}(x - v_\pm t - q_\pm)) + W_0(t)\Phi_\pm,$$

$t \to \pm\infty$, for solutions to (1) with initial states close to a solitary wave. Here $W_0(t)$ is the dynamical group of the free Klein-Gordon equation, $\Phi_\pm$ are the corresponding asymptotic states, and the remainder converges to zero as $t^{-1/2}$ in the "global energy norm" of the Sobolev space $H^1(\mathbb{R}) \oplus L^2(\mathbb{R})$.

Crucial role in the proof play our results on dispersion decay for the corresponding linearized Klein-Gordon equations. Moreover, we introduce a new class of piece-wise quadratic potentials which allow an exact description of the spectral properties for the linearized dynamics.

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Spectral asymptotics for perturbed spherical Schrödinger operators and applications to quantum scattering

Aleksey Kostenko

We find the high energy asymptotics for the singular Weyl–Titchmarsh m-functions and the associated spectral measures of perturbed spherical Schrödinger operators (also known as Bessel operators). We apply this result to establish an improved local Borg–Marchenko theorem for Bessel operators as well as uniqueness theorems for the radial quantum scattering problem with nontrivial angular momentum.

The talk is based on a joint work with G. Teschl.

Γ-hypercyclic set of a bounded linear operator

Bilel Krichen

Based on the study of hypercyclic operators and some spectral tools, we focus in this paper to study some properties of a new set defined by $\Gamma_{hyp}(T) := \{\lambda \in \mathbb{C} \text{ such that } T - \lambda \text{ is hypercyclic}\}$ for a given bounded linear operator T acting on separable Banach space. Several fundamental examples are treated in detail: hyponormal operators, operators satisfying the property (β) and weighted shift on $(l_p, 1 \leq p < \infty)$ spaces. Furthermore, we apply the obtained results to a class of bounded linear operators satisfying $ABA = A^2$ and $BAB = B^2$ and to upper triangular matrix operators.

The talk is based on a joint work with A. Ben Amar, A. Jeribi and E. H. Zerouali.

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The Bessel operator with an inner singularity and Pontryagin spaces

Heinz Langer

We consider the Bessel operator on an interval $[0, a)$ where $a > 1$, and with the singularity at $x = 1$. At this singularity the differential operator is in limit point case from both sides. We describe a general class of interface conditions at $x = 1$ which lead to self-adjoint operators in a Pontryagin space.

Joint work with Malcolm Brown and Matthias Langer.

Spectral estimates and basis properties for block operator matrices

Matthias Langer

In this talk I will discuss estimates for gaps in the essential spectrum of block operator matrices and also estimates for eigenvalues in such gaps. Moreover, I will present results on basis properties of components of eigenvectors corresponding to certain eigenvalues of such block operator matrices. The results are applied to block operator matrices whose entries are differential operators of mixed order.

Variation of discrete spectra of selfadjoint operators in Krein spaces

Leslie Leben

We consider an additive perturbation of a bounded non-negative operator A in a Krein space with a non-negative operator C from a Schatten-von Neumann class of order p, such that $\ker C = \ker C^2$ and 0 is not a singular critical point. We show a qualitative result on the change of the discrete spectrum of A under the perturbation C. More precisely, given an open interval Δ with $0 \notin \overline{\Delta}$, there exist enumerations (α_n) and (β_n) of the discrete eigenvalues of A and of the perturbed operator $A + C$, respectively, such that

$$(\beta_n - \alpha_n) \in \ell^p.$$

The talk is based on a joint work with J. Behrndt and F. Philipp.

On the multiplicity of generalized poles

Annemarie Luger

Generalized poles of generalized Nevanlinna functions are by definition eigenvalues, namely eigenvalues of the self-adjoint relation in a minimal realization of the function.

For scalar generalized Nevanlinna functions Q it is well known that the generalized pole (i.e. eigenvalue) α itself as well as the structure of the algebraic eigenspace can be described in terms of the the asymptotic behavior of Q close to the point α. Basically $-\frac{1}{Q}$ has to vanish of sufficiently high order.

In the case of matrix valued functions the situation is more complicated, e.g. a point α can also be both generalized zero and generalized pole. Here one has to take care also of the fact that the function can have different behavior in different directions. This is done in terms of so-called "pole-cancellation functions". However, this method is rather unpractical and gives only a possibility to characterize the dimension of the non-positive subspace of the algebraic eigenspace, not the whole algebraic eigenspace.

In this talk we are describing how even for matrix valued generalized Nevanlinna functions the asymptotic behavior of a certain transformation of the function can be used in order to describe the structure of the algebraic eigenspace completely.

On completeness of the root vector system of boundary value problem for first order system

Anton Lunyov

We consider first order systems of ordinary differential equations

$$Ly := L(Q)y := -iB\frac{dy}{dx} + Q(x)y = \lambda y, \quad y = \text{col}(y_1, ..., y_n), \qquad (5)$$

subject to the following boundary conditions (BC)

$$Cy(0) + Dy(1) = 0, \qquad C = (c_{jk}), \quad D = (d_{jk}) \in \mathbb{C}^{n\times n}. \qquad (6)$$

Here B is a non-singular diagonal $n \times n$ matrix, and $Q(\cdot)$ is a L^2-potential matrix. We denote by $L_{C,D} := L_{C,D}(Q)$ the operator associated in $L^2([0, 1]; \mathbb{C}^n)$ with the boundary value problem (BVP) (5)–(6).

BVP (5)–(6) has first been investigated by G. D. Birkhoff and R. E. Langer [1]. They introduced the concepts of *regular and strictly regular boundary conditions* (6) and investigated the asymptotic behavior of eigenvalues and eigenfunctions of the corresponding operator $L_{C,D}$. Moreover, they proved *a pointwise convergence result* on spectral decompositions of the operator $L_{C,D}$.

Recently [2] a wider class of boundary conditions, the so-called weakly B-regular BC, was introduced. Moreover, it was shown in [2] that the system of root vectors of BVP (5)–(6) subject to weakly B-regular BC is complete and minimal in $L^2([0,1];\mathbb{C}^n)$.

We obtain some results completing results from [2] for the case of $n\times n$ system. However, the most complete results are obtained for the case of 2×2 Dirac type system ($B = B^*$). Below we present one of the typical result regarding this system.

Let

$$B = \operatorname{diag}(b_1^{-1}, b_2^{-1}), \quad b_1 < 0 < b_2, \quad \text{and} \quad Q = \begin{pmatrix} 0 & Q_{12} \\ Q_{21} & 0 \end{pmatrix}. \tag{7}$$

It is convenient for us to write BC (6) in the following form

$$U_j(y) := a_{j1}y_1(0) + a_{j2}y_2(0) + a_{j3}y_1(1) + a_{j4}y_2(1) = 0, \quad j \in \{1,2\}. \tag{8}$$

Put

$$J_{jk} = \det \begin{pmatrix} a_{1j} & a_{1k} \\ a_{2j} & a_{2k} \end{pmatrix}, \ j,k \in \{1,\dots,4\}.$$

The following statement substantially generalizes Theorem 5.1 from [2].

Theorem. *Let* $Q_{12}(\cdot), Q_{21}(\cdot) \in W_2^{n+1}[0,1]$, *and let* $J_{32} = J_{14} = 0$. *Let also the following conditions be satisfied*

$$\begin{aligned} b_1 J_{13} Q_{12}^{(k)}(0) + (-1)^k b_2 J_{42} Q_{21}^{(k)}(1) &= 0, \quad k \in \{0,1,\dots,n-1\}, \\ b_1 J_{13} Q_{12}^{(n)}(0) + (-1)^n b_2 J_{42} Q_{21}^{(n)}(1) &\neq 0, \end{aligned}$$

and

$$\begin{aligned} b_1 J_{13} Q_{12}^{(k)}(1) + (-1)^k b_2 J_{42} Q_{21}^{(k)}(0) &= 0, \quad k \in \{0,1,\dots,n-1\}, \\ b_1 J_{13} Q_{12}^{(n)}(1) + (-1)^n b_2 J_{42} Q_{21}^{(n)}(0) &\neq 0. \end{aligned}$$

Then the system of root functions of the problem (5),(7),(8) *is complete and minimal in* $L^2\left([0,1];\mathbb{C}^2\right)$.

The talk is based on a joint paper with M. Malamud.

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Spectral theory of Schrödinger operators with infinitely many point interactions and radial positive definite functions

Mark Malamud

Schrödinger operators on the Hilbert space $L^2(\mathbb{R}^d)$, $1 \le d \le 3$, with potentials supported on a discrete (finite or countable) set $X = \{x_j\}_1^m$ of points in $\mathbb{R}^d$ arise in different problems of quantum mechanics (see [1] and references therein). They are given by the formal differential expression

$$\mathfrak{L}_d := \mathfrak{L}_d(X,\alpha) := -\Delta + \sum_{j=1}^{m} \alpha_j \delta(\cdot - x_j), \qquad \alpha_j \in \mathbb{R}, \quad m \in \mathbb{N} \cup \{\infty\}. \quad (9)$$

where $\alpha = \{\alpha_j\}_1^m$ is a sequence of real numbers.

In the case $d = 1$ there are several natural ways to associate the self-adjoint operator (Hamiltonian) on $L^2(\mathbb{R}^1)$ with the differential expression (9). In the case $d = 2, 3$ there is no natural Hamiltonian associated with the differential expression (9) (see [1]).

F. Berezin and L. Faddeev proposed in their pioneering paper [2] to consider the expression (9) (with $m = 1$ and $d = 3$) in the framework of the extension theory of symmetric operators. Following F. Berezin and L. Faddeev one associates the minimal symmetric operator H with the expression (9) as the following restriction of the Laplacian $-\Delta$,

$$H_d := -\Delta \upharpoonright \mathrm{dom} H, \quad \mathrm{dom}(H_d) = \{f \in W^{2,2}(\mathbb{R}^d) : f(x_j) = 0,\ j \in \mathbb{N}\} \quad (10)$$

and studies the spectral properties of *all its selfadjoint extensions.*

The case of $m < \infty$ is well studied. We will discuss the case $m = \infty$. We investigate self-adjoint extensions of the operator $H = H_3$ (realizations of $\mathfrak{L}_3$) in the framework of boundary triplets approach. Namely, we construct a "good" boundary triplet for H_d^* (with $d = 2, 3$) assuming that

$$d_*(X) := \inf_{j \ne k} |x_k - x_j| > 0. \quad (11)$$

Using this boundary triplet Π we parameterize the set of self-adjoint extensions of $H = H_d$, compute the corresponding Weyl function $M(\cdot)$ and investigate various spectral properties of self-adjoint extensions (semi-boundedness, non-negativity, negative spectrum, resolvent comparability, etc.)

Our main result on spectral properties of Hamiltonians with point interactions concerns the absolutely continuous spectrum (*ac*-spectrum). For instance, if

$$C := \sum_{|j-k|>0} \frac{1}{|x_j - x_k|^2} < \infty, \tag{12}$$

we prove that the part $\widetilde{H}E_{\widetilde{H}}(C,\infty)$ of *every self-adjoint extension* $\widetilde{H}$ of H is absolutely continuous. Moreover, under additional assumptions on X, we show that the singular part of $\widetilde{H}_+ := \widetilde{H}E_{\widetilde{H}}(0,\infty)$ is trivial, i.e. $\widetilde{H}_+ = \widetilde{H}_+^{ac}$. In the proof we apply technique elaborated in [3] as well as some new results on the characterization of *ac*-spectrum in terms of the Weyl function.

An important feature of our approach is an appearently new *connection between the spectral theory of operators* (9) *for* $d = 3$ *and the class* Φ_3 *of radial positive definite functions on* $\mathbb{R}^3$. We exploit this connection in both directions.

Namely, we investigate some properties of radial positive definite functions on $\mathbb{R}^d$ that have been inspired by possible applications in the spectral theory of operators (9). If f is such a function and $X = \{x_n\}_1^\infty$ is a sequence of points of $\mathbb{R}^d$, we say that f is *strongly X-positive definite* if there exists a constant $c > 0$ such that for all $\xi_1, \ldots, \xi_m \in \mathbb{C}$,

$$\sum_{j,k=1}^{m} \xi_k \bar{\xi}_j f(x_k - x_j) \geq c \sum_{k=1}^{m} |\xi_k|^2, \qquad m \in \mathbb{N}.$$

Using Schoenberg's theorem we derive a number of results showing under certain assumptions on X that f is strongly X-positive definite and that the Gram matrix $Gr_X(f) := \big(f(|x_k - x_j|)\big)_{k,j\in\mathbb{N}}$ defines a bounded operator on $l^2(\mathbb{N})$. The latter results correlate with the properties of the sequence $\{e^{i(\cdot,x_k)}\}_{k\in\mathbb{N}}$ of exponential functions to form a Riesz-Fischer sequence or a Bessel sequence, respectively, in $L^2(S_r^n;\sigma_n)$ for some $r > 0$.

In particular, using extension theory of the operator H we show that any completely monotone function (in the sense of S, Bernshtein) is X-positive definite provided that $X = \{x_n\}_1^\infty$ satisfy condition (11).

The talk is based on a joint work with Konrad Schmüdgen.

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An extension problem for fractional powers of generators

Pedro José Miana Sanz

In this talk, we extend results of Caffarelli-Silvestre and Stinga-Torrea regarding a characterization of fractional powers of differential operators via an extension problem. Our main result applies to generators A of α-times integrated semigroups, in particular to purely imaginary operators of Schrödinger type and Laplacians defined on Riemannian manifolds or Lie groups. On the way, we give an integral formula for the fractional operator $(-A)^\sigma$, $0 < \sigma < 1$, that generalizes the well-known formula for generators of continuous semigroups. Moreover, a solution to the extension problem can be expressed in terms of the fractional operator. Previous results on the growth of perturbations of α-times integrated semigroups are also improved, that could be of independent interest.

The talk is based on a joint work with José E. Galé and Pablo Raúl Stinga.

On Titchmarsh-Weyl functions and eigenfunction expansions of first-order symmetric systems

Vadim Mogilevskii

Assume that H and $\widehat{H}$ are finite dimensional Hilbert spaces, $\mathbb{H} = H \oplus \widehat{H} \oplus H$, $[\mathbb{H}]$ is the set of all bounded operators in $\mathbb{H}$ and $J \in [\mathbb{H}]$ is a signature operator of the form

$$J = \begin{pmatrix} 0 & 0 & -I_H \\ 0 & iI_{\widehat{H}} & 0 \\ I_H & 0 & 0 \end{pmatrix} : H \oplus \widehat{H} \oplus H \to H \oplus \widehat{H} \oplus H.$$

We will discuss first-order symmetric systems

$$Jy'(t) - B(t)y(t) = \lambda\Delta(t)y + \Delta(t)f(t) \tag{13}$$

with the $[\mathbb{H}]$-valued coefficients $B(t) = B^*(t)$ and $\Delta(t) \geq 0$ defined on an interval $\mathcal{I} = [a, b\rangle$ with the regular endpoint a. In the case $\widehat{H} = \{0\}$ system (1) turns into the Hamiltonian system. We assume the deficiency indices $n_\pm$ of the corresponding minimal relation T_{min} to be arbitrary (possibly unequal).

Our approach is based on the concept of a decomposing boundary triplet for the maximal relation $T_{max}(= T^*_{min})$ introduced in [4] (another construction of a boundary triplet for T_{max} can be found in [1]). This enables us to describe self-adjoint and λ-depending Nevanlinna boundary conditions which are analogs of separated self-adjoint boundary conditions for Hamiltonian systems (1). With a boundary value problem involving such conditions we associate the m-function $m(\cdot) : \mathbb{C}\setminus\mathbb{R} \to [H \oplus \widehat{H}]$. In the case of the Hamiltonian system with equal deficiency indices $n_+ = n_-$ of T_{min} the m-function $m(\cdot)$ coincides with the Titchmarsh-Weyl coefficient. In the simplest case of minimal (unequal) deficiency indices $n_\pm$ the main part of the m-function $m(\cdot)$ coincides with the rectangular Titchmarsh-Weyl coefficient introduced by Hinton and Schneider in [3].

It turns out that $m(\cdot)$ is a Nevanlinna function and its spectral function $\Sigma(\cdot) : \mathbb{R} \to [H \oplus \widehat{H}])$ is a spectral function of the Fourier transform $L^2_\Delta(\mathcal{I}) \ni f \to g_f \in L^2(\Sigma,\, H \oplus \widehat{H})$ with the minimally possible dimension $d = \dim(H \oplus \widehat{H})$. We parametrize all m-functions $m(\lambda)$ (and, consequently, all spectral functions $\Sigma(t)$) in terms of the Nevanlinna boundary parameter at the singular endpoint b. Such a parameterization is given by formula similar to the known Krein formula for resolvents.

Application of these results to differential expressions $l[y]$ of an odd order enables us to complete the results by Everitt and Krishna Kumar [2] on the Titchmarsh-Weyl theory of $l[y]$.

The talk is based on a joint work with S. Albeverio and M. Malamud.

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Spectral gaps of the 1-d Schrödinger operators with periodic distributional potentials

Volodymyr Molyboga

On the complex Hilbert space $L^2(\mathbb{R})$ we study the 1-dimensional Schrödinger operators $\mathrm{S}(q)$ with 1-periodic real-valued distributional potentials $q(x)$:

$$\mathrm{S}(q)u := -u'' + q(x)u, \qquad q(x) = \sum_{k\in\mathbb{Z}} \widehat{q}(k)e^{i\,2k\pi x} \in H^{-1}(\mathbb{T}).$$

The operators $\mathrm{S}(q)$ are well defined on the Hilbert space $L^2(\mathbb{R})$ in the following equivalent different ways: as form-sums, as quasi-differentials ones, as a limit in norm resolvent sense of the sequence of operators with smooth potentials. The operators $\mathrm{S}(q)$ are self-adjoint and lower semibounded. Their spectra are absolutely continuous and have a band and gap structure, see [1] and references therein.

In the talk we discuss the behaviour of the lengths of spectral gaps

$$\gamma_{\mathbf{q}} := \{\gamma_{\mathbf{q}}(n)\}_{n\in\mathbb{N}}$$

of the operators $\mathrm{S}(q)$ in terms of the behaviour of the Fourier coefficients $\{\widehat{q}(k)\}_{k\in\mathbb{Z}}$ of the potentials q with respect to appropriate weight spaces, that is by means of potential regularity [2,3]. We find necessary and sufficient conditions the sequence $\gamma_{\mathbf{q}}$ to be *convergent to zero*, to be *bounded/unbounded*. The talk is based on a joint work with V. Mikhailets.

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The a priori $\tan\Theta$ theorem for spectral subspaces

Alexander Motovilov

Let A be a self-adjoint operator on a separable Hilbert space $\mathfrak{H}$. Assume that the spectrum of A consists of two disjoint components σ_0 and σ_1 such that the set σ_0 lies in a finite gap of the set σ_1. Let V be a bounded self-adjoint operator on $\mathfrak{H}$ off-diagonal with respect to the partition $\mathrm{spec}(A) = \sigma_0 \cup \sigma_1$. It is known that if $\|V\| < \sqrt{2}d$, where $d = \mathrm{dist}(\sigma_0, \sigma_1)$, then the perturbation V does not close the gaps between σ_0 and σ_1 and the spectrum of the perturbed operator $L = A+V$ consists of two isolated components ω_0 and ω_1 originating from σ_0 and σ_1, respectively. We prove that the bound $\|V\| < \sqrt{2}d$ also implies the following (sharp) norm estimate:

$$\|\mathsf{E}_A(\sigma_0) - \mathsf{E}_L(\omega_0)\| \le \sin\left(\arctan\frac{\|V\|}{d}\right),$$

where $\mathsf{E}_A(\sigma_0)$ and $\mathsf{E}_L(\omega_0)$ are the spectral projections of A and L associated with the spectral sets σ_0 and ω_0, respectively.

The talk is based on joint works with S. Albeverio and A. V. Selin.

On the abstract Landauer-Büttiker formula and applications

Hagen Neidhardt

The Landauer-Büttiker formula is an important tool to calculate the current in quantum systems. To give a rigorous proof in a operator-theoretical framework is therefore very important. The talk presents such a proof where in contrast to existing ones the formula is verified for non-semibounded self-adjoint operators. The result is obtained by verifying a Landauer-Büttiker formula for unitary operators. Using the Cayley transform one gets a proof for self-adjoint operators. The result is applied to non-semibounded self-adjoint operators like Dirac operators on the real line and for dissipative Schrödinger operators on bounded intervals.

The talk is based on a common paper with Horia Cornean (Aalborg, Denmark), Lukas Wilhelm (WIAS, Berlin) and Valentin Zagrebnov (Marseille, France).

On the unitary equivalence between differential and finite-difference operators on graphs

Konstantin Pankrashkin

We show that a certain function of the standard Laplacian on any equilateral metric graph is unitary equivalent to the discrete transition operator on the same graph. This allows one to describe some spectral properties of the metric graphs in terms of the combinatorial properties. We will see that this correspondence is just a particular case of a certain general result in the spectral theory of self-adjoint extensions in terms of boundary triples and generalizes some results of the inverse spectral theory for scalar-type Weyl functions proved in a paper by Albeverio, Brasche, Malamud, Neidhardt in 2005.

Classification of non-smooth pseudodifferential operators

Christine Pfeuffer

R. Beals and J. Ueberberg proved a classification of pseudodifferential operators with smooth symbols. Since non-smooth pseudodifferential operators are sometimes used in order to calculate the regularity of a partial differential equation, such a classification would also be useful in this case. Therefore we will show, that every linear operator, which satisfies some specific continuity assumptions, is a non-smooth pseudodifferential operator of the class $C^\tau S^m_{1,0}(\mathbb{R}^n \times \mathbb{R}^n)$. Analogously to the proof of J. Ueberberg in the smooth case, one reduces this statement to the following: Each linear operator T, which fulfills some specific continuity properties, is a non-smooth pseudodifferential operator of the class $C^\tau S^0_{0,0}(\mathbb{R}^n \times \mathbb{R}^n)$. The main new difficulty is to take care of the limited mapping properties of pseudodifferential operators with non-smooth symbols.
The talk is based on a part of my PhD-thesis advised by Prof. H. Abels.

Eigenvalues in gaps of selfadjoint operators in Pontryagin spaces

Friedrich Philipp

Given two selfadjoint operators A and B in a Hilbert space with n-dimensional resolvent difference, i.e.

$$\dim \operatorname{ran} \left((A-i)^{-1} - (B-i)^{-1} \right) = n,$$

and a gap Δ in the essential spectrum of A (and thus also of B), it is well known that

$$|\operatorname{eig}(A,\Delta) - \operatorname{eig}(B,\Delta)| \le n,$$

where $\operatorname{eig}(\cdot,\Delta)$ denotes the number of eigenvalues (counting multiplicities) of the respective operator in Δ. In this talk we present a result which generalizes this theorem to Pontryagin spaces. To be precise, if A and B are selfadjoint operators in a Pontryagin space $(\Pi_\kappa, [\cdot,\cdot])$ with n-dimensional resolvent difference and Δ is gap in the essential spectra of A and B, we prove that

$$|\operatorname{sig}(\mathcal{L}_\Delta(A)) - \operatorname{sig}(\mathcal{L}_\Delta(B))| \le n,$$

where $\operatorname{sig}(\mathcal{M})$ denotes the signature difference of the inner product $[\cdot,\cdot]$ on the subspace $\mathcal{M}$, and $\mathcal{L}_\Delta(\cdot)$ is the spectral subspace of the respective operator corresponding to Δ. In particular, this implies that

$$|\operatorname{eig}(A,\Delta) - \operatorname{eig}(B,\Delta)| \le n + 2\kappa.$$

The latter estimate is sharp, as will be shown by means of simple examples.

We apply the general result to a class of eigenvalue depending boundary value problems for Sturm-Liouville operators.

The talk is based on joint work with J. Behrndt.

Boundary pairs and Dirichlet-to-Neumann operators

Olaf Post

We present a concept for defining the Dirichlet-to-Neumann (DtN) operator in a purely funcional-analytic framework starting with a closed form in a Hilbert space and a bounded operator from the form domain into an auxiliary Hilbert space. The basic example we have in mind is the energy form on a manifold with (possibly non-smooth) boundary. We present Krein-like resolvent formulas, spectral relations between the associated "Neumann"-Operator and the DtN operator and illustrate the ideas with many examples.

Inverse spectral problems for Dirac operators with summable matrix-valued potentials

Dmytro Puyda

We solve the direct and inverse spectral problems for self-adjoint Dirac operators T_q generated by the differential expressions

$$\mathfrak{t}_q := \frac{1}{\mathrm{i}} \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \frac{\mathrm{d}}{\mathrm{d}x} + \begin{pmatrix} 0 & q \\ q^* & 0 \end{pmatrix}$$

and the boundary conditions $y_1(0) = y_2(0)$, $y_1(1) = y_2(1)$. Here q is an $r \times r$ matrix-valued function with entries belonging to $L_p(0,1)$, $p \in [1,\infty)$, and I is the identity $r \times r$ matrix.

Namely, the spectrum of the operator T_q consists of countably many isolated real eigenvalues of finite multiplicity, accumulating only at $+\infty$ and $-\infty$. We denote by $\lambda_j(q)$, $j \in \mathbb{Z}$, the pairwise distinct eigenvalues of the operator T_q labeled in increasing order so that $\lambda_0(q) \le 0 < \lambda_1(q)$. Further, let m_q stand for the Weyl-Titchmarsh function of the operator T_q. The function m_q is an $r \times r$ matrix-valued meromorphic Herglotz function and $\{\lambda_j(q)\}_{j\in\mathbb{Z}}$ is the set of its poles. We set

$$\alpha_j(q) := - \operatorname*{res}_{\lambda=\lambda_j(q)} m_q(\lambda), \qquad j \in \mathbb{Z},$$

and call $\alpha_j(q)$ the *norming matrix* of the operator T_q corresponding to the eigenvalue $\lambda_j(q)$.

The sequence $\mathfrak{a}_q := ((\lambda_j(q), \alpha_j(q)))_{j\in\mathbb{Z}}$ will be called the *spectral data* of the operator T_q, and the matrix-valued measure

$$\mu_q := \sum_{j=-\infty}^{\infty} \alpha_j(q)\delta_{\lambda_j(q)},$$

where δ_λ is the Dirac delta-measure centered at the point λ, will be called its *spectral measure*. We give a complete description of the class of the spectral data for the operators under consideration (which is equivalent to description of the class of the spectral measures), show that the spectral data determine the operator uniquely and suggest an efficient method for reconstructing the operator from the spectral data.

The talk is based on a joint work with Ya. Mykytyuk.

Existence of maximal semidefinite invariant subspaces and semigroup properties of some classes of ordinary differential operators

Sergey Pyatkov

We examine differential operators of the form

$$Lu = \frac{1}{g(x)} L_0 u, \quad x \in (a,b), \tag{1}$$

where L_0 is an ordinary differential operator of order $2m$ defined by the differential expression

$$L_0 u = \sum_{i,j=0}^{m} \frac{d^i}{dx^i} a_{ik}(x) \frac{d^j u}{dx^j} \quad (x \in (a,b)) \tag{2}$$

and the boundary conditions

$$B_k u = \sum_{i=0}^{2m-1} (\alpha_{ik} u^{(i)}(a) + \beta_{ik} u^{(i)}(b)) = 0 \ (k = 1, 2, \ldots, 2m), \tag{3}$$

where we should cancel the corresponding summands if $b = +\infty$ or $a = -\infty$. It is possible that $(a,b) = \mathbb{R}$. The real-valued function $g(x)$ in (1) changes its sign on (a,b). We assume that the operator L is J-dissipative in the Krein space $F_0 = L_{2,g}(a,b)$, where an inner product and an indefinite inner product are defined by the equalities

$$(u,v)_0 = \int_a^b |g(x)| u(x) \overline{v(x)}\, dx, \quad [u,v]_0 = \int_a^b g(x) u(x) \overline{v(x)}\, dx.$$

It is possible that $0 \in \sigma(L)$, i. e., we consider the singular case too. Under certain additional conditions on the behavior of the function $g(x)$, we prove that there exist maximal semidefinite invariant subspaces $H^{\pm}$ of L such that $F_0 = H^+ + H^-$ (the sum is direct) and the restrictions $\pm L|_{H^{\pm}}$ are the generators of analytic semigroups.

The finite section method for infinite Vandermonde matrices

André Ran

The finite section method for infinite Vandermonde matrices is the focus of this talk. More precisely, we shall consider an infinite system of equations of the form $Ax = d$, where

$$A = \begin{pmatrix} 1 & 1 & 1 & \dots \\ a_0 & a_1 & a_2 & \dots \\ a_0^2 & a_1^2 & a_2^2 & \dots \\ \vdots & \vdots & \vdots & \end{pmatrix}, \qquad d = \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \end{pmatrix}.$$

Assuming that all a_j's are different, the $n \times n$ finite sections A_n of the matrix A will all be invertible. Hence for each n there is a unique solution of $A_n x_n = P_n d$, where P_n is the projection onto the first n coordinates. Embedding x_n into an infinite vector by continuing with zeros, we can study the question whether in some topology these vectors converge to a solution of $Ax = d$.

It will be shown that for a large class of infinite Vandermonde matrices the finite section method indeed converges in l_1 sense if the right hand side d of the equation is in a suitably weighted $l_1(\alpha)$ space. More explicit results are obtained for a wide class of examples.

This is joint work with András Serény.

Spectral analysis and representations of solutions of abstract integro-differential equations in a Hilbert space

Nadezda Rautian

We obtain and study the representations of the solutions of certain classes of abstract integro-differential equations in a Hilbert space as a series of exponentials on the base of the structure and asymptotic of spectra of the operator-valued functions which are the symbols of these equations. These integro-differential equations arise in the theory of heat propagation in media with memory, viscoelasticity etc.

Spectral measures of Jacobi operators with random potentials

Rafael del Rio

Let H_ω be a self-adjoint Jacobi operator with a potential sequence $\{\omega(n)\}_n$ of independently distributed random variables with continuous probability distributions and let μ_ϕ^ω be the corresponding spectral measure generated by H_ω and the vector ϕ. We consider sets $\mathcal{A}(\omega)$ which depend on ω, but are independent of two consecutive given entries of the secuence ω, and prove that $\mu_\phi^\omega(\mathcal{A}(\omega)) = 0$ for almost every ω. This result is applied to show equivalence relations between spectral measures for random Jacobi matrices and to study the interplay of the eigenvalues of these matrices and their submatrices.

This is based on joint work with Luis Silva.

Dynamical localization for Delone-Anderson operators

Constanza Rojas-Molina

Delone-Anderson operators are used to study a particle moving in a medium with impurities that have a quasi-crystalline spatial configuration. A particular feature of quasi-crystals is the lack of translation invariance, which yields a break of ergodicity in the model. We prove suitable Wegner estimates and initial length scale estimates using, and emphasize the role of quantitative unique continuation principles in these estimates. As a consequence, we obtain dynamical localization at the bottom of the spectrum and a bound on the size of the localization interval in terms of the geometric parameters of the underlying Delone set. We conclude our study with a discussion on the existence of the integrated density of states for these models in the framework of randomly coloured Delone sets. This is joint work with F. Germinet and P. Müller.

The operator div A grad for sign-indefinite A

Stephan Schmitz

Differential expressions of the type div$A(\cdot)$grad with sign-indefinite coefficient matrix $A(\cdot)$ arise in the modelling of metamaterials with negative refraction index.

Using the representation theory for indefinite quadratic forms, we prove, that these differential expressions define self-adjoint operators on bounded domains.

The spectral asymptotics and solvability of boundary value problems for this operator is discussed as well.

This talk is based on joint work with A. Hussein, V. Kostrykin and D. Krejčiřík.

Existence of the integrated density of states for Hamiltonians on Cayley graphs

Fabian Schwarzenberger

In this talk we study the existence of the integrated density of states (IDS) for operators defined on Cayley graphs. The IDS of a selfadjoint operator encodes the distribution of its spectrum on the real axis. There are two generic ways to define the IDS. The first one is to state it as a so-called Pastur-Shubin trace formula as a rather abstract object. In the second way one considers finite dimensional approximants and states the IDS as limit of distribution functions. In this talk we focus on the latter way. The operators in question are Hamiltonians defined on geometric structures given via finitely generated groups. These geometries include

$\mathbb{Z}^d$ and k-valent trees but do go far beyond. Depending on the specific properties of the groups, we can show weak convergence as well as uniform convergence. Beside this we are able to treat random models given via a percolation process. Involved methods are ergodic theory, large deviation theory and the investigation of tiling properties of groups.

The talk is based on joint works with S. Ayadi, D. Lenz, F. Pogorzelski, S. Schumacher and I. Veselić.

Generators with a closure relation

Felix Schwenninger

Suppose that a block operator of the form $\begin{pmatrix} & A_1 \\ A_2 & 0 \end{pmatrix}$, acting on the Banach space $X_1 \times X_2$, generates a contraction C_0-semigroup. We show that the operator A_S defined by $A_S x = A_1 \begin{pmatrix} x \\ SA_2 x \end{pmatrix}$ with the natural domain generates a contraction semigroup on X_1. Here, S is a boundedly invertible operator for which $-S^{-1} + \epsilon \mathrm{I}$ is dissipative for sufficiently small ϵ. With the result the existence and uniqueness of solutions of the heat equation can be derived from the wave equation.

On an estimate in the subspace perturbation problem

Albrecht Seelmann

We study the problem of variation of spectral subspaces for linear self-adjoint operators under an additive perturbation. The aim is to find the best possible upper bound on the norm of the difference of two spectral projections associated with isolated parts of the spectrum of the perturbed and unperturbed operators in terms of the strength of the perturbation.

In our approach, we formulate a constrained optimization problem on a finite set of parameters, whose solution gives an estimate on the norm of the difference of the corresponding spectral projections. In particular, this estimate is stronger than the one recently obtained by Albeverio and Motovilov in [arXiv:1112.0149v2 (2011)].

Approximation of spectra and pseudospectra of bounded linear operators on Banach spaces

Markus Seidel

It is well known that spectra of operators do not behave continuously with respect to small perturbations. When trying to approximate an operator A or its spectrum by finite matrices (e.g. finite sections of A) the situation becomes even worse.

One possible way to avoid this problem is to slightly change the notions and to consider ϵ-pseudospectra instead. These substitutes are known to provide a significantly better continuity but of course at the price of being different from the spectrum. In 2008, Hansen introduced the so called (N, ϵ)-pseudospectra, which share the continuity with the ϵ-pseudospectra, but also mimic the spectrum in a sense. Moreover, he showed that this concept leads to suitable approximations for the spectrum of Hilbert space operators which only require the consideration of finite matrices.

The aim of this talk is to demonstrate what obstacles occur in the Banach space case, and how Hansens concept and its crucial results can be successfully generalized beyond the comfortable world of Hilbert spaces.

Existence and set of solutions of the Riemann–Hilbert boundary value problem with a two - side curling at infinity of the order less than 1

Pavel Shabalin

We obtain a solution to the Riemann–Hilbert boundary value problem in the theory of analytic functions on the half-plane in the case that the coefficients of the boundary conditions have discontinuity points of the first kind and boundary value problem has two - side curling at infinity of the order . We derive formulas for the general solution and investigate the pictures of solvability of these problems.

The talk is based on a joint work with R. Salimov

Uniqueness for inverse Sturm–Liouville problems using three spectra with a finite number of transmission conditions

Mohammad Shahriari

We establish various uniqueness results for inverse spectral problems of Sturm–Liouville operators using three spectra with a finite number of discontinuities at interior points at which we impose the usual transmission conditions. We consider both the cases of classical Robin and of eigenparameter dependent boundary conditions.

Spectral portraits of non-self-adjoint Sturm-Liouville operators with small physical parameter

Andrei Shkalikov

We deal with spectral problems of the form $-i\varepsilon y'' + q(x)y = \lambda y$ where λ and ε are the spectral and physical parameters, respectively. The potential $q(x)$ is assumed to be analytic with some additional properties. The case $q(x) = -q(-x),\ x \in (a,a) \subseteq \mathbb{R}$ corresponds to the so-called PT-symmetric Sturm-Liouville operators. The aim is to understand the spectrum behavior as $\varepsilon \to 0$. It turns out that the spectrum concentrates along some critical curves in the complex plane which form "the limit spectral graph". The problem is to understand the geometry of this graph, to find analytic formulae describing its parts and to write the asymptotic eigenvalue distribution (uniformly, as $\varepsilon \to 0$ along the limit spectral curves. All these problems will be discussed as well as applications in hydrodynamics.

Spectral multiplicity of Schrödinger operators on star-graphs

Sergey Simonov

We consider the Schrödinger operator on a star-graph with n finite or infinite edges and the standard interface condition at the origin. We are interested what the local spectral multiplicity of this operator is. The answer is expected in terms of the spectral measures μ_l, $l = 1, ..., n$, of the Schrödinger operators on the edges with Dirichlet boundary condition at the origin. For $n = 2$ such a situation is described by Kac theorem. It says in particular that the singular spectrum of the operator on the whole graph (just the real line in that case) is always simple. We will discuss a generalisation and refinement of Kac result for the case $n > 2$. The answer depends on the type of the spectrum, i.e., is different for its absolutely continuous and singular parts with respect to the Lebesgue measure and to the measure $\mu = \sum_{l=1}^{n} \mu_l$. In particular, the multiplicity of the singular spectrum cannot exceed $(n-1)$, but can be smaller.

The talk is based on a joint work with H. Woracek.

Passive scattering systems

Olof Staffans

There is an extensive literatur on a class of linear time-invariant dynamical systems called "well-posed scattering passive systems". Such a system is generated by an operator S which is called a scattering passive system node. In the existing literature such a node is typically introduced by first giving a long list of assumptions which imply that S is a system node, and then adding an inequality which forces this system node to be scattering passive. Here we proceed in the opposite direction: we start by requiring that S satisfies the appropriate inequality, and then ask the question of what additional conditions are needed in order for S to be a system node. The answer is surprisingly simple: A necessary and sufficient condition for an operator S to be a scattering passive system node is that S is closed and maximal within the class of operators that satisfy the appropriate passivity inequality. In the absense of external inputs and outputs this condition is identical to the standard condition which characterizes the class of operators which generate contraction semigroups on some Hilbert space.

Extinction of solution of semilinear parabolic equations with degenerate absorption potential

Kateryna Stiepanova

The poster is devoted to the behavior of energy (generalized) solutions for a wide class of semilinear parabolic equations. We investigate a model Cauchy-Neumann problem for parabolic equations of non-stationary diffusion-semilinear absorption with a degenerate absorption potential. More precisely, the following problem is considered:

$$\left(|u|^{q-1}u\right)_t - \sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(|\nabla_x u|^{q-1}\frac{\partial u}{\partial x_i}\right) + a_0(x)|u|^{\lambda-1}u = 0 \text{ in } \Omega \times (0,T), \tag{1}$$

$$\frac{\partial u}{\partial n}\Big|_{\partial\Omega\times[0,T]} = 0, \tag{2}$$

$$u(x,0) = u_0(x), \quad x \in \Omega. \tag{3}$$

Here $q > 1$, $0 < \lambda < 1$, and $a_0(x) \geq 0$ is an arbitrary continuous function. The initial function $u_0(x)$ is from $L_2(\Omega)$, where $\Omega \subset \mathbb{R}^N (N \geqslant 1)$ is a bounded domain with C^1 - boundary. The origin belongs to Ω $(0 \in \Omega)$.

The main focus of our study is the long-time extinction property for solutions to the initial-boundary problem (1), (2), (3). We obtain a sharp condition on the degeneration of the potential $a_0(x)$ that guarantees the long-time extinction.

Let $a_0(x)$ be a potential satisfying the inequality

$$a_0(x) \geq c_0 \exp\Big(-\frac{\omega(|x|)}{|x|^{q+1}}\Big), \quad x \in \Omega \setminus \{0\},$$

where $c_0 > 0$ is a constant, and $\omega(\cdot)$ is an arbitrary function such that

(A) $\omega(\tau) > 0$ $\forall \tau > 0$, (B) $\omega(0) = 0$, (C) $\omega(\tau) \to 0$ as $\tau \to 0$ monotone.

Theorem. *Let $u_0(x) \in L_2(\Omega)$. Let $\omega(\cdot)$ be a continuous nondecreasing function that satisfies assumptions* (A), (B), (C) *and the following main condition:*

$$\int_0^c \frac{\omega(\tau)}{\tau} d\tau < \infty.$$

Suppose also that $\omega(\cdot)$ *satisfies the technical condition*

$$\frac{\tau\,\omega'(\tau)}{\omega(\tau)} \le 1-\delta \qquad \forall\,\tau\in(0,\tau_0),\quad \tau_0>0,\quad 0<\delta<1.$$

Then an arbitrary energy solution $u(x,t)$ *of problem* (1), (2), (3) *vanishes on* Ω *in a finite time* $T<\infty$.

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Spectral properties of quasicrystal Laplacians.

Peter Stollmann

Quasicrystals are known for their peculiar properties and it is generally accepted that certain strange phenomena should occur in the spectral decomposition of the respective Laplacians. These are modeled with the help of certain dynamical systems. The talk is based on joint work with S. Klassert, D. Lenz and C. Seifert and gives an introductory account of the rigorous results and the open questions in this area of Mathematical Physics.

On matrices with the dominant main diagonal

Liudmila Sukhocheva

Let $\mathcal{H}$ be a Hilbert space with the scalar product $(\cdot,\cdot)$. A bounded everywhere defined linear operator $A:\mathcal{H}\to\mathcal{H}$ is called an operator with the dominant main diagonal if for any orthonormal basis $\{e_j\}_{j=1}^N$, $N \leq \infty$, the following inequality hold:

$$|(Ae_j, e_j)| \geq \sum_{k=1,\, k\neq j}^{N} |(Ae_j, e_k)|, \quad j = \overline{1, N}.$$

For a selfadjoint operator A a necessary and sufficient condition when this operator has the dominant main diagonal is given.

The research was supported by grant 12-01-00102-a of the RFBR.

Characterization of stability of contractions

Attila Szalai

We characterize those sequences $\{h_n\}_{n=1}^{\infty}$ of bounded analytic functions, which have the property that an absolutely continuous contraction T is stable (that is the powers T^n converge to zero) exactly when the operators $h_n(T)$ converge to zero in the strong operator topology. Our result is extended to polynomially bounded operators too.

The talk is based on a joint work with L. Kérchy.

Localization for non-monotone Anderson-type models

Martin Tautenhahn

We consider random Hamiltonians on the lattice given by

$$H_\omega = -\Delta + V_\omega, \quad V_\omega(x) = \sum_{k\in\mathbb{Z}^d} \omega_k u(x-k), \tag{14}$$

on $\ell^2(\mathbb{Z}^d)$, whose randomness is generated by a sign-indefinite single-site potential $u \in \ell^1(\mathbb{Z}^d; \mathbb{R})$ with i.i.d. random variables ω_k, $k \in \mathbb{Z}^d$. The main challenge of models of the type (14) is that u may change its sign. As a consequence, certain properties of H_ω depend in an non-monotone way on the random parameters ω_k, $k \in \mathbb{Z}^d$. For this reason many established tools for the spectral analysis of random Schrödinger operators are not directly applicable in this model.

In this talk we discuss recent results on localization in the large disorder regime for via the fractional moment method. The specific assumption on the single-site potential u is that $\operatorname{supp} u$ is finite and has fixed sign on the boundary of its support.

Moreover, we present a Wegner estimate for exponentially decaying single-site potentials, i.e. $|u(k)| \leq C\mathrm{e}^{-\alpha\|x\|_1}$ but not necessarily of bounded support, which is valid on the whole energy axis. A Wegner estimate is an upper bound on the expected number of eigenvalues in some energy interval $[a, b]$ of a finite volume restriction H_Λ of H_ω, $\Lambda \subset \mathbb{Z}^d$. Since our Wegner bound is valid on the whole energy axis, it can be used for a localization proof via multiscale analysis in any energy region where the initial length scale estimate holds.

The talk is based on joint works with A. Elgart, N. Peyerimhoff and I. Veselić

Convergence analysis of high-order time-splitting pseudo-spectral methods for a class of nonlinear Schrödinger equations

Mechthild Thalhammer

In this talk, the issue of favourable numerical methods for the space and time discretisation of low-dimensional nonlinear Schrödinger equations is addressed. The objective is to provide a stability and error analysis of high-accuracy discretisations that rely on spectral and splitting methods. As a model problem, the time-dependent Gross–Pitaevskii equation arising in the description of Bose–Einstein condensates is considered. For the space discretisation pseudo-spectral methods collocated at the associated quadrature nodes are analysed. For the time integration high-order exponential operator splitting methods are studied, where the decomposition of the function defining the partial differential equation is chosen in accordance with the underlying spectral method. The convergence analysis relies on a general framework of abstract nonlinear evolution equations and fractional power spaces defined by the principal linear part. Essential tools in the derivation of a temporal global error estimate are further the formal calculus of Lie-derivatives and bounds for iterated Lie-commutators. Numerical examples for higher-order time-splitting pseudo-spectral methods applied to time-dependent Gross–Pitaevskii equations illustrate the theoretical result.

A Schauder and Riesz basis criterion for non-self-adjoint Schrödinger operators with periodic and antiperiodic boundary conditions

Vadim Tkachenko

Under the assumption that $V \in L^2([0,\pi];dx)$, we derive necessary and sufficient conditions for (non-self-adjoint) Schrödinger operators $-d^2/dx^2 + V$ in $L^2([0,\pi];dx)$ with periodic and antiperiodic boundary conditions to possess a Riesz basis of root vectors, i.e., eigenvectors and generalized eigenvectors spanning the range of the Riesz projection associated with the corresponding periodic and antiperiodic eigenvalues.

We also discuss the case of a Schauder basis for periodic and antiperiodic Schrödinger operators $-d^2/dx^2 + V$ in $L^p([0,\pi];dx)$, $p \in (1,\infty)$.

This is a joint work with Fritz Gesztesy. The detailed exposition of our results has been recently published in *Journal of Differential Equations*, 253 (2012), 400-437.

Spectral enclosures for block operator matrices

Christiane Tretter

In this talk various recent results on spectral enclosures for block operator matrices will be presented. Block operator matrices occur frequently in mathematical physics, e.g. when considering systems of differential equations. In the non-selfadjoint case, it is particularly important to establish analytic enclosures for eigenvalues and other parts of the spectrum since then numerical computations are prone to be unreliable.

On a class of boundary control systems

Sascha Trostorff

We study a class of abstract boundary control systems in a Hilbert space setting, employing the theory of extrapolation spaces for boundedly invertible operators. We show the well-posedness of the problem and state a criterion for the conservativity of the system. The theory will be exemplified by a boundary control problem for Maxwell's equations.

The talk is based on a joint work with R. Picard and M. Waurick.

Spectral analysis and correct solvability of abstract integro-differential equations in a Hilbert space

Victor Vlasov

We obtain the correct solvability of abstract integro-differential equations in a Hilbert space. Then we study the spectra of the operator-valued functions which are the symbols of above mentioned equations. We analyze the integro-differential equations arising in applications (Gurtin-Pipkin type equations describing the process of heat propagation in media with memory, integro-differential equations arising in the theory of viscoelasticity).

Large time behaviour of heat kernels and admissible potentials

Hendrik Vogt

Let T be a positivity improving selfadjoint C_0-semigroup on $L_2(\Omega, \mu)$ with generator $-H$. The following two questions are going to be addressed in the talk:

- Assuming the semigroup operators $T(t)$ have integral kernels p_t, what is the long time behaviour of $p_t(x, y)$, given $x, y \in \Omega$?
- For a measurable potential $V\colon \Omega \to [0, \infty)$, when does the initial value problem

  $$u'(t) + Hu(t) = Vu(t) \quad (t > 0), \qquad u(0) = u_0$$

  have a positive *exponentially bounded* solution, given a positive initial value $u_0 > 0$?

The talk is based on joint work with M. Keller, D. Lenz and R. Wojciechowski.

Resonant delocalization: the phase diagram of the Anderson model on trees

Simone Warzel

Motivated by the quest for a theory of quantum transport in disordered media, in 1958 P.W. Anderson came up with a model for a quantum particle in a random energy landscape. Among its interesting features is a conjectured sharp transition from a regime of localized eigenstates to one of diffusive transport. Until today it remains a mathematical challenge to establish these features in the framework of random Schrödinger operators. In this talk, I will describe recent progress in the understanding of the spectral and dynamical properties of such operators in case the underlying configuration space is hyperbolic such as a tree graph. Among the surprising phenomena which we discover is that even at weak disorder the regime of diffusive transport extends well beyond the one of the graph Laplacian into the regime of Lifshitz tails. As will be explained in the lecture, the mathematical mechanism for the appearance of conducting states in this non-perturbative regime are disorder-induced resonances.

A Hilbert space perspective on ordinary differential equations with memory term.

Marcus Waurick

We discuss ordinary differential equations with delay and memory terms in Hilbert spaces. By introducing a time derivative as a normal operator in an appropriate Hilbert space, we develop a new approach to a solution theory covering integro-differential equations, neutral differential equations and general delay differential equations within a unified framework.

The talk is based on a joint work with A. Kalauch, R. Picard, S. Siegmund, and S. Trostorff.

High- and low- energy analysis and Levinson's theorem for the selfadjoint matrix Schrödinger operator on the half line

Ricardo Weder

The matrix Schrödinger equation with a selfadjoint matrix potential is considered on the half line with the general selfadjoint boundary condition at the origin. When the matrix potential is integrable, the high-energy asymptotics are established for the related Jost matrix, the inverse of the Jost matrix, and the scattering matrix. Under the additional assumption that the matrix potential has a first moment, it is shown that the scattering matrix is continuous at zero energy. An explicit formula is provided for the scattering matrix at zero energy. The small-energy asymptotics are established also for the related Jost matrix, its inverse, and various other quantities relevant to the corresponding direct and inverse scattering problems. Furthermore, Levinson's theorem is derived, relating the number of bound states to the change in the argument of the determinant of the scattering matrix.

The talk is based on a joint work with T. Aktosun and M. Klaus.

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Extremal sectorial operators and relations

Henrik Winkler

A (linear) relation S in a Hilbert space H is said to be *sectorial* with vertex at the origin and semiangle α, $\alpha \in [0, \pi/2)$, if

$$|\Im(\varphi', \varphi)| \leq (\tan\alpha)\Re(\varphi', \varphi), \quad \{\varphi, \varphi'\} \in S.$$

A sectorial relation S in a Hilbert space H is said to be *maximal sectorial* if the existence of a sectorial relation T in H with $S \subset T$ implies $S = T$. Sectorial operators or relations have maximal sectorial extensions. In this case S decomposes into the orthogonal sum of a densely defined maximal sectorial operator and a purely multivalued part. A maximal sectorial extension H of S is said to be *extremal* if

$$\inf\{\, \Re(h' - \varphi', h - \varphi) : \{\varphi, \varphi'\} \in S \,\} = 0 \quad \text{for all} \quad \{h, h'\} \in H.$$

The Friedrichs extension and the Kreĭn-von Neumann extension are extremal in this sense. It is shown that all extremal extensions of a sectorial relation can be characterized in terms of factorizations. As in the case of nonnegative relations, the factorizations of the Kreĭn-von Neumann and Friedrichs extensions lead to a novel approach to the transversality and equality of the extreme extensions, and to the notion of positive closability, meaning that the Kreĭn-von Neumann extension is an operator.

The talk is based on a joint work with S. Hassi, A. Sandovici and H. de Snoo.

On the spectrum of waveguides in planar photonic bandgap structures

Ian Wood

We study a Helmholtz-type spectral problem related to the propagation of electromagnetic waves in photonic crystal waveguides. The waveguide is created by introducing a linear defect into a two-dimensional periodic medium. The defect is infinitely extended and aligned with one of the coordinate axes. The perturbation is expected to introduce guided mode spectrum inside the band gaps of the fully periodic, unperturbed spectral problem. We prove that guided mode spectrum can be created by arbitrarily small perturbations and that, after performing a Floquet decomposition in the axial direction of the waveguide, for any fixed value of the quasi-momentum the perturbation generates at most finitely many new eigenvalues inside the gap.

Hamiltonians and Riccati equations for unbounded control and observation operators

Christian Wyss

We consider the control algebraic Riccati equation

$$A^*X + XA - XBB^*X + C^*C = 0$$

for the case that A is a normal operator with compact resolvent, $B \in L(U, H_{-s})$ and $C \in L(H_s, Y)$, $0 \le s \le 1$. Here $H_s \subset H \subset H_{-s}$ are the usual fractional domain spaces corresponding to A. This setting includes the case where B and C correspond to point or boundary control and observation, respectively. We show the existence of infinitely many solutions X of the Riccati equation using invariant subspaces of the Hamiltonian operator matrix

$$T = \begin{pmatrix} A & -BB^* \\ -C^*C & -A^* \end{pmatrix}.$$

Just like in the finite-dimensional case, each such solution corresponds to a certain choice of eigenvalues of T. We also obtain conditions for bounded, nonnegative, and nonpositive solutions. Our main tools are Riesz bases of eigenvectors of T and indefinite inner products.

The talk is based on joint work with B. Jacob and H. J. Zwart.

Extremal L^1 problem for entire functions and spectral theory for canonical systems

Peter Yuditskii

The following is the *résumé* of Akhiezer's paper *Uzagal'nennya odniei mimimum-zadachi Korkina-Zolotareva*:

A solution in elliptic functions is given for the following problem: among all polynomials of the form

$$P_n(x) = x^n + p_1 x^{n-1} + \cdots + p_n$$

find the one for which the value

$$\int_{-1}^{\alpha} |P_n(x)| dx + \int_{\beta}^{1} |P_n(x)| dx$$

assumes its minimum with the fixed α and β ($-1 < \alpha < \beta < 1$).

It does not seem obvious that the famous Akhiezer's polynomials orthogonal on two intervals were first constructed in this paper (at least the *résumé* does not contain any hint in this direction). We discuss a similar problem in the classes of entire functions and explain its connections with the spectral theory for canonical systems and the Direct Cauchy Theorem in Widom domains.

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Friedrichs extension for operators associated with symplectic systems (and their special cases)

Petr Zemanek

The theory of the *Friedrichs extension* goes back to J. von Neumann and it was shown by Friedrichs in 1934 that for a symmetric densely defined linear operator $\mathcal{L}$ which is bounded below in a Hilbert space H, there exists at least one self-adjoint extension of the minimal operator associated with $\mathcal{L}$ with the same lower bound.

In this talk, we present recent progress achieved in this field with a focus on operators connected with some special cases of symplectic systems (more specifically, with the linear Hamiltonian differential system and with any even order Sturm–Liouville dynamic equation on a time scale), see [1, 2, 3]. We also discuss an open problem concerning the Friedrichs extension for operators associated with a symplectic difference system.

References

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List of participants

Abdelmoumen, Boulbeba .... Boulbeba.Abdelmoumen@ipeis.rnu.tn
Adamyan, Vadym .... vadamyan@onu.edu.ua
Ammann, Kerstin .... kerstin.ammann@univie.ac.at
Arendt, Wolfgang .... wolfgang.arendt@uni-ulm.de
Artamonov, Nikita .... nikita.artamonov@gmail.com
Bátkai, András .... batka@cs.elte.hu
Behrndt, Jussi .... behrndt@tugraz.at
Belyi, Sergey .... sbelyi@troy.edu
Brown, Malcolm .... Malcolm.Brown@cs.cardiff.ac.uk
Cheremnikh, Evgen .... echeremn@polynet.lviv.ua
Christodoulides, Yiannis .... eng.cy@fit.ac.cy
Eckhardt, Jonathan .... jonathan.eckhardt@univie.ac.at
Elst, Tom ter .... terelst@math.auckland.ac.nz
Exner, Pavel .... exner@ujf.cas.cz
Fellner, Klemens .... klemens.fellner@uni-graz.at
Förster, Karl-Heinz .... foerster@math.tu-berlin.de
Freiberg, Uta .... freiberg@mathematik.uni-siegen.de
Geher, Gyoergy .... gehergyuri@gmail.com
Gernandt, Hannes .... hannes.gernandt@tu-ilmenau.de
Gheondea, Aurelian .... gheondea@jot.theta.ro
Gilbert, Daphne .... daphne.gilbert@dit.ie
Goloshchapova, Nataliia .... natalygoloshchapova@gmail.com
Grubb, Gerd .... grubb@math.ku.dk
Grunert, Katrin .... katring@math.ntnu.no
Haenel, Andre .... andre.haenel@mathematik.uni-stuttgart.de
Hansmann, Marcel .... marcel.hansmann@mathematik.tu-chemnitz.de
Haskovec, Jan .... jan.haskovec@kaust.edu.sa
Hassi, Seppo .... sha@uwasa.fi
Kaltenbaeck, Michael .... michael.kaltenbaeck@tuwien.ac.at
Kats, Boris .... katsboris877@gmail.com
Katsnelson, Victor .... victor.katsnelson@weizmann.ac.il
Keller, Matthias .... m.keller@uni-jena.de
Kerchy, Laszlo .... kerchy@math.u-szeged.hu
Komech, Alexander .... alexander.komech@univie.ac.at
Kopylova, Elena .... elena.kopylova@univie.ac.at
Kostenko, Aleksey .... duzer80@gmail.com
Krichen, Bilel .... krichen_bilel@yahoo.fr
Kühn, Christian .... kuehn@tugraz.at
Langer, Heinz .... hlanger@mail.zserv.tuwien.ac.at

Langer, Matthias .... m.langer@strath.ac.uk
Leben, Leslie .... leslie.leben@tu-ilmenau.de
Lotoreichik, Vladimir .... vladimir.lotoreichik@gmail.com
Luger, Annemarie .... luger@math.su.se
Lunyov, Anton .................................A.A.Lunyov@gmail.com
Malamud, Mark .... mmm@telenet.dn.ua
Miana Sanz, Pedro José .... pjmiana@unizar.es
Mogilevskii, Vadim .... vim@mail.dsip.net
Molyboga, Volodymyr .... vm.imath@gmail.com
Motovilov, Alexander.......................... motovilv@gmail.com
Nasseri, Amir Bahman........................ amirbahman@hotmail.de
Neidhardt, Hagen .... neidhard@wias-berlin.de
Pankrashkin, Konstantin...konstantin.pankrashkin@math.u-psud.fr
Pfeuffer, Christine ....
................Christine.Pfeuffer@mathematik.uni-regensburg.de
Philipp, Friedrich .... philipp@math.tu-berlin.de
Post, Olaf .... post@math.hu-berlin.de
Puyda, Dmytro .... dpuyda@gmail.com
Pyatkov, Sergey .... pyatkov@math.nsc.ru
Ramos, Alberto Gil Couto Pimentel.................................
.... a.g.c.p.ramos@maths.cam.ac.uk
Ran, André .... a.c.m.ran@vu.nl
Rautian, Nadezda .... nrautian@mail.ru
Rio, Rafael del .... delriomagia@gmail.com
Rohleder, Jonathan....................... rohleder@math.tugraz.at
Rojas-Molina, Constanza........................ crojasm@u-cergy.fr
Romero, Natalia .... natalia.romero@unirioja.es
Schmitz, Stephan .... schmist@uni-mainz.de
Schwarzenberger, Fabian ....
..............fabian.schwarzenberger@mathematik.tu-chemnitz.de
Schwenninger, Felix................... f.l.schwenninger@utwente.nl
Seelmann, Albrecht .... seelmann@mathematik.uni-mainz.de
Seidel, Markus .... markus.seidel@mathematik.tu-chemnitz.de
Shabalin, Pavel .... pavel.shabalin@mail.ru
Shahriari, Mohammad .... mohamad.shahriari@yahoo.com
Shkalikov, Andrei .... ashkalikov@yahoo.com
Simonov, Sergey .... sergey.a.simonov@gmail.com
Snoo, Henk de............................... desnoo@math.rug.nl
Staffans, Olof .... olof.staffans@abo.fi
Stautz, Marko .... m.stautz@tu-bs.de
Stiepanova, Kateryna........................ kitti_dob@rambler.ru

Stollmann, Peter......peter.stollmann@mathematik.tu-chemnitz.de
Sukhocheva, Liudmila........................l.suhocheva@yandex.ru
Szalai, Attila.............................szalaiap@math.u-szeged.hu
Tautenhahn, Martin ....
....................martin.tautenhahn@mathematik.tu-chemnitz.de
Teschl, Gerald ....Gerald.Teschl@univie.ac.at
Thalhammer, Mechthild ....mechthild.thalhammer@uibk.ac.at
Tkachenko, Vadim.........................tkachenk@math.bgu.ac.il
Tretter, Christiane..........................tretter@math.unibe.ch
Trostorff, Sascha ....sascha.trostorff@tu-dresden.de
Vlasov, Victor ....vikvvlasov@rambler.ru
Vogt, Hendrik...........................vogt@math.tu-clausthal.de
Warzel, Simone ....warzel@ma.tum.de
Waurick, Marcus ....marcus.waurick@tu-dresden.de
Weder, Ricardo..weder@unam.mx
Winkler, Henrik ....henrik.winkler@tu-ilmenau.de
Wood, Ian ....i.wood@kent.ac.uk
Woracek, Harald......................harald.woracek@tuwien.ac.at
Wyss, Christian.......................wyss@math.uni-wuppertal.de
Yuditskii, Peter............................petro.yudytskiy@jku.at
Zemanek, Petr ....zemanekp@math.muni.cz