36) a) $\int x^2 e^{-x^3} dx =$

subs $y = -x^3 \Rightarrow dy = -3x^2 dx$

$= -\int \frac{x^2}{3x^2} \cdot e^y dy = -\frac{1}{3} e^y + c = \underline{\underline{-\frac{1}{3} e^{-x^3} + c}}$

b) $(a \neq 0)$ $\int \frac{1}{\sqrt{a^2+x^2}} dx = \int \frac{1}{\sqrt{a^2(1+\frac{x^2}{a^2})}} dx =$

subs $y = \frac{x}{a}$

$\Rightarrow dy = \frac{1}{a}$

$= a \cdot \int \frac{1}{\sqrt{a^2} \cdot \sqrt{1+y^2}} dy = \operatorname{sgn}(a) \cdot \int \frac{1}{\sqrt{1+y^2}} dy$

$= \operatorname{sgn}(a) \operatorname{arsinh}(y) + c = \underline{\underline{\operatorname{sgn}(a) \operatorname{arsinh}\left(\frac{x}{a}\right) + c}}$

c) $\int \frac{1}{\sqrt{4x+x^2}} dx = \int \frac{1}{\sqrt{x}\sqrt{4+x}} dx =$

subs $y = \frac{\sqrt{x}}{2}$

$\Rightarrow dy = \frac{1}{4\sqrt{x}} dx$

$= 4 \cdot \int \frac{1}{\sqrt{4+4y^2}} dy = 2 \cdot \int \frac{1}{\sqrt{1+y^2}} dy =$

$= 2 \cdot \operatorname{arsinh}(y) + c = \underline{\underline{2 \cdot \operatorname{arsinh}\left(\frac{\sqrt{x}}{2}\right) + c}}$

$a = 0$: $\int \frac{1}{\sqrt{x^2}} dx = \ln(|x|)$

d) $\int x\sqrt{1+x}\, dx = \int (y-1)\sqrt{y}\, dy = \int y^{\frac{3}{2}} dy - \int y^{\frac{1}{2}} dy =$

subs $y = 1+x \Rightarrow dy = dx$

$= \frac{2}{5} y^{\frac{5}{2}} - \frac{2}{3} y^{\frac{3}{2}} + c = \underline{\underline{\frac{2}{5}(1+x)^{\frac{5}{2}} - \frac{2}{3}(1+x)^{\frac{3}{2}} + c}}$

37a) $\int \frac{x^3+5x^2-4x+2}{x^2+7x+12}\,dx := I$

(i) Polynomdivision:

$(x^3+5x^2-4x+2) : (x^2+7x+12) = x-2$
$-(x^3+7x^2+12x)$
$-2x^2-16x+2$
$-(-2x^2-14x-24)$
$-2x+26$

$\Rightarrow I = \int x-2\,dx + \int \frac{2x-26}{x^2+7x+12}\,dx$

NST von $x^2+7x+12$: $x_{12} = \frac{-7\pm\sqrt{49-48}}{2} = \frac{-7\pm 1}{2} = -4, -3$

PBZ: $\frac{A}{x+4} + \frac{B}{x+3} = \frac{2x-26}{(x+4)(x+3)}$

$\Leftrightarrow (x+3)A + (x+4)B = 2x-26$

$\Rightarrow A+B = 2$
$\Rightarrow 3A+4B = -26$

$\Rightarrow 3A+4(2-A) = 8-A = -26 \Leftrightarrow A = \underline{\underline{34}} \Rightarrow B = -\underline{\underline{32}}$

$\Rightarrow I = \frac{x^2}{2} - 2x + 34 \cdot \ln(|x+4|) - 32 \cdot \ln(|x+3|) + c$

37) b)

$$\int \frac{x^2+3x+7}{(x+2)(x^2+6x+10)} dx := I$$

PBZ: $\frac{A}{(x+2)} + \frac{Bx+C}{(x^2+6x+10)} = \frac{x^2+3x+7}{(x+2)(x^2+6x+10)}$

$\Leftrightarrow (x^2+6x+10)A + (x+2)(Bx+C) = x^2+3x+7$

$\Rightarrow A + B = 1$

$6A + 2B + C = 3$

$10A + 2C = 7$

$\Rightarrow 6A + 2(1-A) + C = 4A + C + 2 = 3$

$\Leftrightarrow 4A + C = 1$

$\Rightarrow 10A + 2C = 10A + 2(1-4A) = 2A + 2 = 7$

$\Rightarrow A = \frac{5}{2}$

$\rightarrow B = -\frac{3}{2}$

$\Rightarrow C = -9$

$\Rightarrow I = \frac{5}{2} \int \frac{1}{x+2} dx + \int \frac{-\frac{3}{2}x - 9}{x^2+6x+10} dx \overset{VO}{=}$

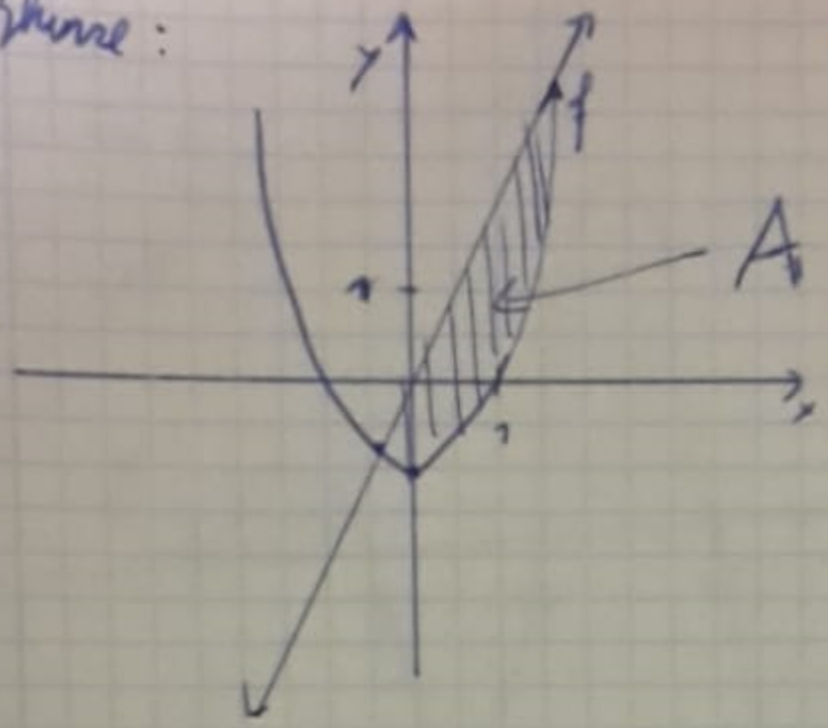
$= \frac{5}{2} \ln(|x+2|) - \frac{3}{4} \ln(|x^2+6x+10|) +$

$+ \left(-9 + \frac{3}{4} \cdot 6\right) \frac{1}{\sqrt{10-9}} \arctan\left(\frac{x+3}{\sqrt{10-9}}\right) + c =$

$= \frac{5}{2} \ln(|x+2|) - \frac{3}{4} \ln(|x^2+6x+10|) +$

$- \frac{9}{2} \arctan(x+3) + c$

38) Skizze:



Schnittpunkte: $f(x) = g(x) \Leftrightarrow 2x = x^2 - 1 \Leftrightarrow x^2 - 2x - 1 = 0$

$$\Rightarrow x_{1,2} = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\cdot\sqrt{2}}{2} = \underline{\underline{1 \pm \sqrt{2}}}$$

in $[1-\sqrt{2}, 1+\sqrt{2}]$ gilt $f(x) \geq g(x)$, Sei $a = 1-\sqrt{2}$
$b = 1+\sqrt{2}$

$$\Rightarrow A = \int_{1-\sqrt{2}}^{1+\sqrt{2}} (f-g)(x)\,dx$$

$$\Rightarrow A = \int_a^b 2x - x^2 + 1\,dx = [x^2]_a^b - \left[\frac{x^3}{3}\right]_a^b + [x]_a^b =$$

$$= (1+\sqrt{2})^2 - (1-\sqrt{2})^2 - \frac{(1+\sqrt{2})^3}{3} + \frac{(1-\sqrt{2})^3}{3} +$$

$$+ (1+\sqrt{2}) - (1-\sqrt{2}) =$$

$$= \cancel{1} + 2\cdot\sqrt{2} + \cancel{2} - \cancel{1} + 2\cdot\sqrt{2} - \cancel{2} +$$

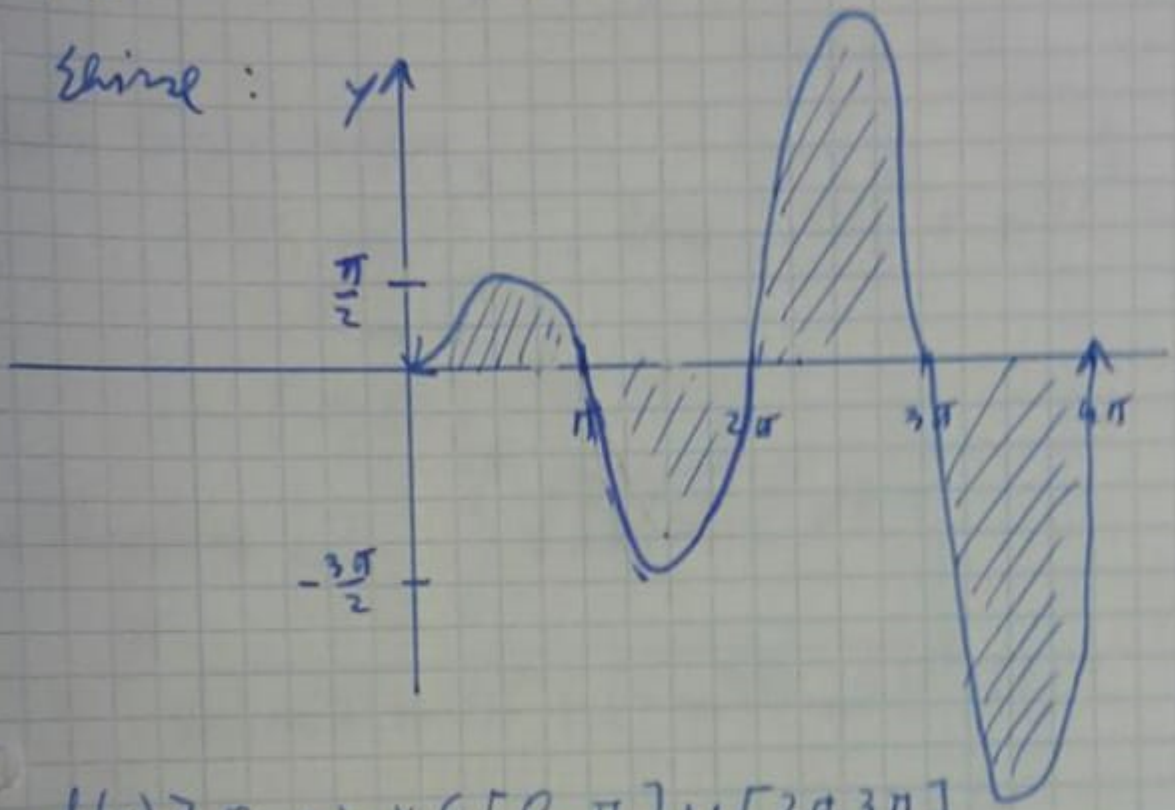
$$-\left(\frac{\cancel{1^3} + 3\cdot\sqrt{2} + \cancel{3\cdot 2} + \sqrt{2}^3}{3}\right) + \left(\frac{\cancel{1^3} - 3\cdot\sqrt{2} + \cancel{3\cdot 2} - \sqrt{2}^3}{3}\right)$$

$$+ \cancel{1} + \sqrt{2} - \cancel{1} + \sqrt{2} = 4\cdot\sqrt{2} - 2\cdot\sqrt{2} + 2\cdot\sqrt{2} - \frac{2}{3}\cdot\sqrt{2}^3$$

$$= 4\cdot\sqrt{2} - \frac{4}{3}\sqrt{2} = \underline{\underline{\frac{8}{3}\sqrt{2}}}$$

39) $f(x) = x \cdot \sin x \quad , \quad 0 \leq x \leq 4\pi$

Skizze:



$f(x) \geq 0 \Leftrightarrow x \in [0, \pi] \cup [2\pi, 3\pi]$

$f(x) \leq 0 \Leftrightarrow x \in [\pi, 2\pi] \cup [3\pi, 4\pi]$

$$\Rightarrow A = \int_0^{\pi} f(x)\,dx - \int_{\pi}^{2\pi} f(x)\,dx + \int_{2\pi}^{3\pi} f(x)\,dx - \int_{3\pi}^{4\pi} f(x)\,dx$$

Berechne $\int x \cdot \sin(x)\,dx = -x \cdot \cos(x) + \int \cos(x)\,dx =$

$$= \sin(x) - x \cdot \cos(x) + C$$

$$\Rightarrow A = \left[\sin(x) - x \cdot \cos(x)\right]_0^{\pi} - \left[\sin(x) - x \cdot \cos(x)\right]_{\pi}^{2\pi} +$$

$$+ \left[\sin(x) - x \cdot \cos(x)\right]_{2\pi}^{3\pi} - \left[\sin(x) - x \cdot \cos(x)\right]_{3\pi}^{4\pi} =$$

$$= \pi - 0 - (-2\pi - \pi) + (3\pi + 2\pi) - (-4\pi - 3\pi) =$$

$$= \pi + 3\pi + 5\pi + 7\pi = \underline{\underline{16\pi}}$$

40)

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1, \quad -1 \le x \le 1$$
$$-1 \le y \le 1$$

Wähle Parametrisierung

$x(t) = \cos(t)^3$ Zeige $x(t)^{\frac{2}{3}} + y(t)^{\frac{2}{3}} = \cos(t)^2 + \sin(t)^2 = 1$ ✓

$y(t) = \sin(t)^3$

$t \in [0, 2\pi)$

$\Rightarrow$ (i) Bogenlänge $L = \int_0^{2\pi} \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2}\, dt =$

$$= \int_0^{2\pi} \sqrt{(-3\cos^2(t)\sin(t))^2 + (3\sin^2(t)\cos(t))^2}\, dt =$$

$$= 3 \cdot \int_0^{2\pi} \sqrt{(\cos^2(t)\sin^2(t))\underbrace{(\cos^2(t) + \sin^2(t))}_{=1}}\, dt =$$

$$= 3 \cdot \int_0^{2\pi} \sqrt{\cos^2(t)\sin^2(t)}\, dt =$$

es reicht 1. Quadrant 4 Mal zu berechnen, dort gilt $\sin(t) \ge 0$, $\cos(t) \ge 0$

$$= 3 \cdot 4 \cdot \int_0^{\frac{\pi}{2}} \cos(t)\sin(t)\, dt$$

Berechne $\int \cos(t)\sin(t)\, dt \underset{\text{part. Integration}}{=} \sin(t)\sin(t) - \int \sin(t)\cos(t)\, dt$

$\Rightarrow \frac{\sin^2(t)}{2} = \int \cos(t)\sin(t)\, dt$

$\Rightarrow L = 3 \cdot 4 \cdot \left[\frac{\sin^2(t)}{2}\right]_0^{\frac{\pi}{2}} = 6 \cdot 1 = \underline{\underline{6}}$

(ii) von Kurve eingeschlossene Fläche:

$$A = 4 \cdot \int_0^{\frac{\pi}{2}} - y(t) \cdot \dot{x}(t)\, dt$$

da jeder Viertelbogen gleiche Fläche einschließt

und $x(t)$ monoton fallend ist

$$\Rightarrow A = 4 \cdot \int_0^{\frac{\pi}{2}} + \sin(t)^3\, 3 \cdot \cos(t)^2 \sin(t)\, dt =$$

$$= 4 \cdot 3 \cdot \int_0^{\frac{\pi}{2}} \sin^4(t) \cos^2(t)\, dt$$

Berechne $\int \sin^4(t)\cos^2(t)\, dt = \int \sin^4(t)(1 - \sin^2(t))\, dt =$

$$= -\int \sin^6(t)\, dt + \int \sin^4(t)\, dt$$

$$\int \sin^4(t)\, dt \underset{\text{part. Int}}{=} -\sin^3(t)\cos(t) + 3 \cdot \int \sin^2(t) \cos^2(t)\, dt =$$

$$= -\sin^3(t)\cos(t) + 3 \cdot \int \sin^2(t)(1 - \sin^2(t))\, dt =$$

$$= -\sin^3(t)\cos(t) + 3 \cdot \int \sin^2(t)\, dt - 3 \cdot \int \sin^4(t)\, dt$$

$$\Rightarrow \int \sin^4(t)\, dt = \frac{1}{4}\left(-\sin^3(t)\cos(t) + 3 \cdot \int \sin^2(t)\, dt\right)$$

$$\int \sin^6(t)\, dt \underset{\text{part. Int.}}{=} -\sin^5(t)\cos(t) + 5 \cdot \int \sin^4(t)(1 - \sin^2(t))\, dt$$

$$= -\sin^5(t)\cos(t) + 5 \int \sin^4(t)\, dt - 5 \int \sin^6(t)\, dt$$

$$\Rightarrow \int \sin^6(t)\, dt = \frac{1}{6}\left(-\sin^5(t)\cos(t) + 5 \int \sin^4(t)\, dt\right)$$

Berechne $\int \sin^2(t)\,dt = -\sin(t)\cos(t) + \int \cos(t)\,dt =$

$$= -\sin(t)\cos(t) + t - \int \sin^2(t)\,dt$$

$$\Rightarrow \int \sin^2(t)\,dt = \frac{1}{2}\left[-\sin(t)\cos(t) + t + c\right]$$

$$\Rightarrow \int \sin^4(t)\,dt = -\frac{\sin^3(t)\cos(t)}{4} + \frac{3}{8}\left[-\sin(t)\cos(t) + t\right] + c$$

$$\Rightarrow \int \sin^6(t)\,dt = \frac{1}{6}\left(-\sin^5(t)\cos(t)\right)$$

$$-\frac{5}{4\cdot 6}\sin^3(t)\cos(t) - \frac{5\cdot 3}{4\cdot 6}\,\frac{1}{2}\sin(t)\cos(t) - \frac{5\cdot 3\cdot 1}{4\cdot 6\cdot 2}\,t + c$$

$$\Rightarrow A = -12\cdot\int_0^{\frac{\pi}{2}} \sin^6(t)\,dt + 12\cdot\int_0^{\frac{\pi}{2}} \sin^4(t)\,dt =$$

$\sin(0)=0$
$\cos\left(\frac{\pi}{2}\right)=0$
$\Rightarrow \sin(t)\cdot\cos(t)=0$
für $t\in\{0,\frac{\pi}{2}\}$

$$= -\cancel{12}^{\,3}\cdot\frac{\cancel{15}^{\,5}}{\cancel{48}_{\cancel{16}\,4}}\,\frac{\pi}{2} + \frac{\cancel{12}^{\,3}\cdot 3}{\cancel{8}_{\,2}}\,\frac{\pi}{2} =$$

$$= -\frac{15}{4}\,\frac{\pi}{2} + \frac{9}{2}\,\frac{\pi}{2} = \frac{18-15}{4}\,\frac{\pi}{2} = \underline{\underline{\frac{3}{8}\pi}}$$