

Depth First Search (DFS)

①

Input $G=(V,E)$ (a graph with its vertex set V , its edge set E specified in terms of the set of neighbors $N(v)$, $\forall v \in V$)

Output: $DFSNum(v)$, $seen(v)$, $prec(v)$, $\forall v \in V$ (*)

Main Code

Begin

```
1 for  $v \in V$  do  $DFSNum(v) := 0$ 
1 for  $v \in V$  do  $seen(v) := FALSE$ 
1 for  $v \in V$  do  $prec(v) := v$ 
1  $t := 0$ 
1  $seen(s) := TRUE$ 
1  $DFS(G, s)$ 
End (main)
```

Procedure $DFS(G, v)$

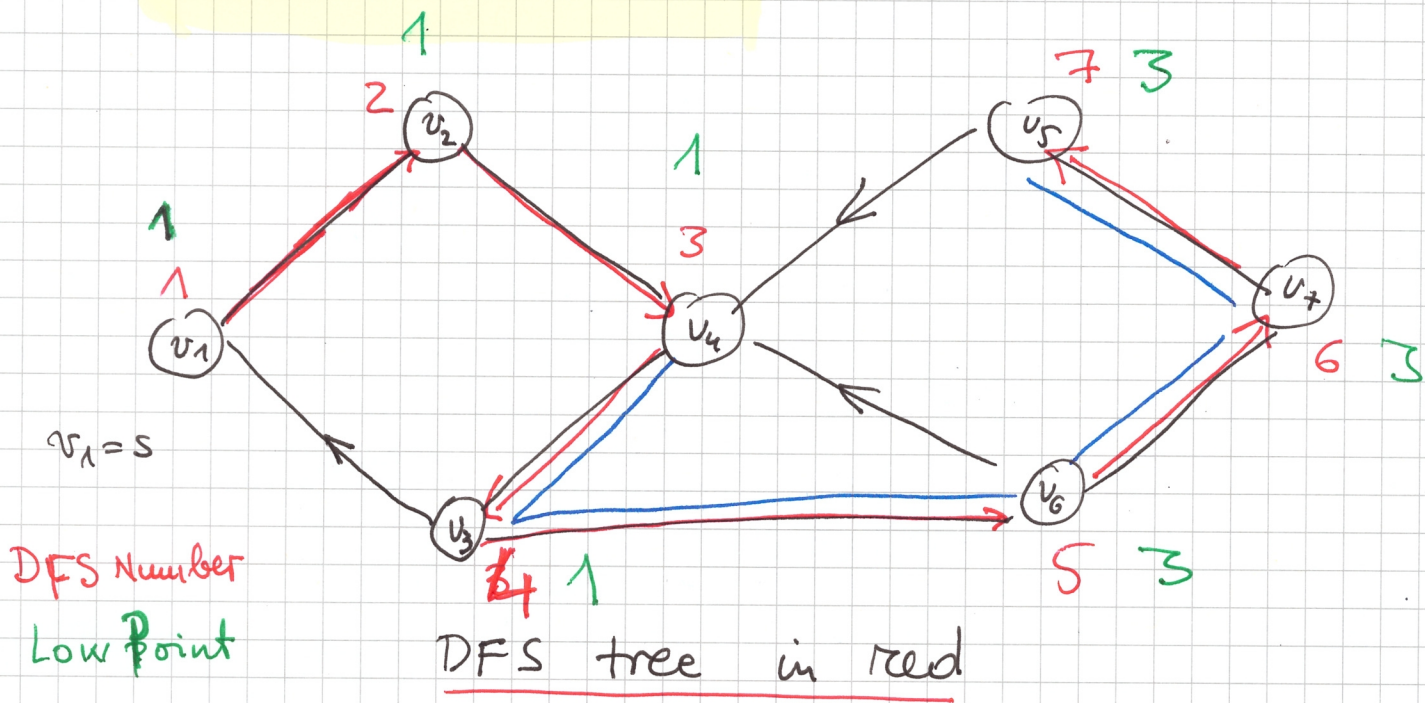
Begin

```
1  $t := t + 1$ 
1  $DFSNum[v] := t$ 
1 for  $w \in N(v)$  do
1   if  $seen(w) = FALSE$  then
1      $seen(w) := TRUE$ ;  $prec(w) := v$ ;
1      $DFS(G, w)$ 
1   endif
1 endfor
End DFS
```

(*) $DFSNum(v)$ specifies the order in which vertices are discovered during DFS
 $seen(v)$ specifies for each vertex whether it has been seen during DFS
 $prec(v)$ specifies the predecessor of vertex v during DFS

Example 1 DFS

(2)



- Tree-edges in red; Backward edges in black (with direct.)
- leading edges: only $\{v_1, v_3\}$
- Illustration of Lemma 1: $w = v_3$
 $T' = T_{\setminus v_3} + \{v_4, v_3\}$, i.e. the blue path

Application of Theorem 1

(b) Since there is only one leading edge,
all tree-edges belong to the same block

(a) every backward edge belongs to the same block
as the tree edge ending at the start vertex
of the backward edge.

Thus just one block in this case $\Rightarrow k(G) \geq 2$

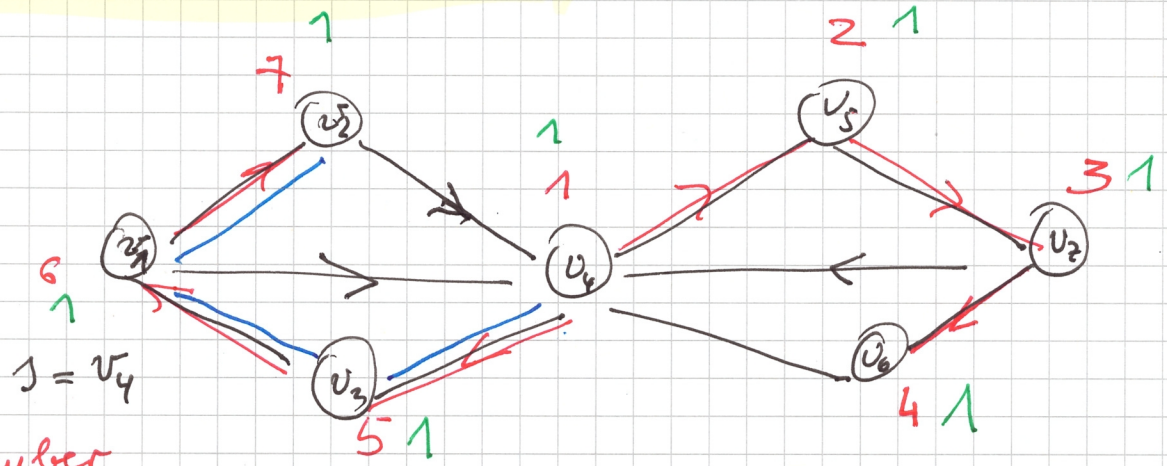
Illustration of Theorem 2

$s = v_1$ is not a cut-vertex; there is just one leading
edge incident to v_1

Any $v \neq v_1$ is not a cut vertex because the
only leading edge starts at v_1 . So there are no cut-vertices

Example 2 DFS

(3)



DFS Number

low Point

- Tree edges in red ; Backward edges in black (with direction)
- leading edges : $\{v_4 v_5\}$ and $\{v_4 v_3\}$
- Illustration of Lemma 1 $w = v_3$ $T' = T_{v_3} + \{v_4 v_3\}$
i.e. the blue path
- Application of Theorem 1:
 - (a) $\{v_4, v_5\}, \{v_5, v_2\}, \{v_2, v_6\}$ belong to the same block B_1
 $\{v_4, v_3\}, \{v_3, v_1\}, \{v_1, v_2\}$ belong to the same block B_2
 - (a) Backward edges $\{v_2, v_4\}, \{v_6, v_4\}$ belong to B_1
 $\{v_1, v_4\}, \{v_2, v_4\}$ belong to B_2

Application of Theorem 2

s is a cut vertex because there are 2 leading edges starting at s $\Rightarrow G$ is not 2-connected, $K(G) \leq 1$.
 $\forall v \neq s$, v is not a cut-vertex because all leading edges start at s

bc(G)

