Game Theory Summer term 2018

Exercise Sheet 2

11. The Troll and Traveller game

    Solve the Troll and Traveller game (cf. lecture) on the network represented in Figure 1, i.e. determine the value of the game and specify a (mixed) Nash eequilibrium for each of the players. Recall that a mixed strategy of each player corresponds to a probability distribution over the s-t-paths in the network. Assume that the toll to be payed on each arc equals 1 unit.

12. The Bomber and Battleship game

    At this class of games a battleship (player II) moves along a uniform one-dimensional grid i.e. the integer lattice $\mathbb{Z}$, and is initially located at the origin. At each time-step in $\{0, 1, 2 \ldots\}$ the battleship moves either one unit to the left or one unit to the right in the grid and it then remains there until the next time-step. A bomber (player I) who can see the current location of the battleship drops a bomb at some time j, $j \in \{0, 1, 2 \ldots\}$ over some site in $\mathbb{Z}$. The bomb arrives at time $j + 2$ and destroys the battleship if it fits it. (The battleship cannot see the bomber or the bomb in time to change course.) Denote by G_n the version of the game where the bomber has enough fuel to drop her bomb at any time in $\{0, 1, \ldots, n\}$. Set up the payoff matrix for the game and compute its value in the following cases

    (a) $n = 0$, i.e. for the game G_0,

    (b) $n = 1$, i.e. for the game G_1,

    (c) $n = 2$, i.e. for the game G_2.

    Assume in all cases that both players choose their strategy for the whole duration of the game at time 0, thus neither the bomber nor the battleship are allowed to take any decision fter the time 0.

13. The Cheetahs and Antelopes game

    Two cheetahs each chase one of the three antelopes. If they catch the same one, they have to share it. The antelopes are Large, Small, and Tiny, and their values to the cheetahs are l, s and t, respectively. Write the 3×3 matrix for this game. Assume that $t < s < l < 2s$ and that
    $$\frac{l}{2}\frac{2l - s}{s + l} + s\frac{2s - l}{s + l} < t\,.$$
    Find the pure equilibria and the symmetric mixed equilibria.

14. Consider a symmetric two-player general-sum game with payoffs given as follows, where $a, b, c, d \in \mathbb{R}$.

    Player II

		S1	S2
	S1	(a,a)	(b,c)
Player I	S2	(c,b)	(d,d)

    A *max-min strategy* or *an optimal safety strategy* of player I is a strategy $\bar{x}$ for which the maximum of $\min_{y \in \Delta_2} x^t Ay$ is achieved, i.e. $\bar{x} \in \text{argmax}\{\min_{y \in \Delta_2} x^t Ay \colon x \in \Delta_1\}$, where
    $$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Analogously a *max-min strategy* or *an optimal safety strategy* of player II is a strategy $\bar{y}$ for which the maximum of $\min_{x\in\Delta_1} x^t By$ is achieved, i.e. $\bar{y} \in \operatorname{argmax}\{\min_{x\in\Delta_1} x^t By\colon y \in \Delta_2\}$, where $B = A^t$.

(a) Compute optimal safey strategies for both players and show that in general they do not build a Nash equilibrium. Assume for example $a < c < d < b$.

(b) Compute a mixed Nash equilibrium and show that it results in the same player payoffs as the optimal security strategies, respectively.

15. Consider the following n-person general sum game. Each person writes down an integer between 1 and 100, on a separate slip of paper, alongside his name. The game-master then reads the numbers on each slip of paper and calculates the average x of all the numbers written by the players. The winner of the game is the player or the players who wrote down the number that is closest to x. The winners equally divide the prize (for example 1 unit) between them. Specify the utility function of each player in this game and give a Nash equilibrium.

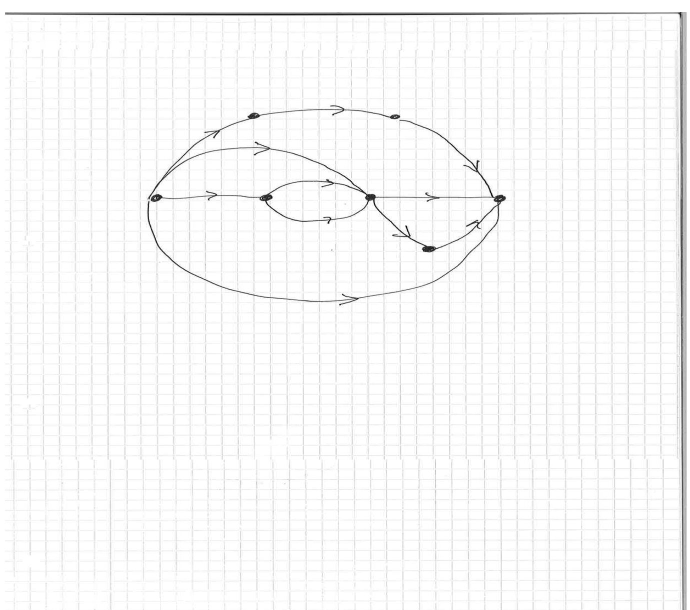

Figure 1: Network for Exercise No. 11