Game Theory, Summer Term 2018
Exercise Sheet 6

37. Brent and Stuart are the only bidders in a sealed-bid first-price auction of a medical device. Brent knows that Stuart's private value is uniformly distributed over $[0, 2]$ and that Stuart's strategy is $\beta(v) = \frac{v^2}{3} + \frac{v}{3}$.

    (a) What is Brent's optimal strategy?

    (b) What is Brent's expected payment if he implements this optimal strategy (as a function of his own private value)?

38. (a) Suppose that the private values of two bidders in a sealed-bid first-price auction are independent and uniformly distributed over the set $\{0, 1, 2\}$. In other words, each bidder has three possible equally probable private values. The bids in the auctions must be nonnegative integers. Find all the equilibria.

    (b) Find all the quilibria under the same assumptions when the auction is a sealed-bid second-price auction.

    (c) Compare the auctioneer's expected revenues under both auction methods.

39. Show that the three-bidder single-item auction in which the item is allocated to the highest bidder at a price equal to the third highest bid is not truthful. Would truthfulness lead to a Nash equilibrium in this case?

40. Consider the *(sealed-bid) k-unit Vickrey auction* in which the auctioneer has k identical items, each bidder wants only one of them, and the top k bidders win the auction at a price equal to the $(k + 1)$-st highest bid. Prove that this auction is truthful.

41. Consider a Vickrey second-price auction with two bidders. Show that for each choice of bidder 1's value v_1 and any possible bid $b \neq v_1$ he might submit, there is a bid by the other bidder that yields bidder 1 strictly less utility than he would had gotten had he bid truthfully.

42. *All-pay auction*

    This is a single-item n-bidder auction, $n \in \mathbb{N}$, in which the item is allocated to the player that bids the highest and <u>every</u> player is charged his bid. Thus in this kind of auction $p_i(v_i) = \beta_i(v_i)$ holds for every player $i \in \{1, 2, \ldots, n)$ with strategy β_i, private value v_i, and payment $p(v_i)$. For example, architects competing for a construction project submit design proposals. While only one architect wins the contest, all competitors expend the effort to prepare their proposals. Thus the participants need to make the strategic decision as to how much effort to put in.

    Assume that the private values V_i, $1 \leq i \leq n$, are independently, identically distributed random variables with a uniform distribution over $[0, 1]$. Show that the only symmetric increasing equilibrium $(\beta, \beta, \ldots, \beta)$ with a strictly increasing function β on $[0, 1]$ is given by $\beta(v) := \frac{n-1}{n} v^n$.

43. *War of attrition auction*

    This auction allocates to the highest bidder, charges him the second-highest bid, and charges any other bidder his bid, respectively. For example animals fighting over territory expend energy. A winner emerges when the fighting ends, and each animal has expended energy up to the point at which it dropped out, or in the case of the winner, until he was the last one left. Assume that the bids are committed to up-front, rather than in the more natural setting where a player's bid (which corresponds to the decision about how long to stay in) can be adjusted over the course of the auction. Apply the revenue equivalence theorem (and its corollary) to determine a symmetric equilibrium $(\beta, \ldots, \beta)$ with β being strictly increasing in this case. Assume that the values of all players are identically, independently distributed with a strictly distribuion differentiable function F.