Combinatorial optimization 1
Winter term 2016/2017

5th working sheet

34. Let G be a digraph with edge capacities $u\colon E(G) \to \mathbb{R}_+$ and balance values $b\colon V(G) \to \mathbb{R}$ such that $\sum_{v \in V(G)} b(v) = 0$. Show that there exists a b-flow in (G, u, b) iff the following inequality holds for all $X \subseteq V(G)$:
$$\sum_{e \in \delta^+(X)} u(e) \geq \sum_{v \in X} b(v)\,.$$
Hint: In order to show the sufficient condition the characterization of the existence of a b-flow by means of the maximum flow problem can be used. Then apply the max-flow-min-cut-theorem.

35. (a) Solve the minimum cost flow problem for the network represented in Figure 1 by means of the minimum-cycle-cancelling-algorithm. The pairs of numbers next to the edges represent the capacities and the costs, respectively, the left-most number being the capacity and the right-most number being the cost. The numbers next to the vertices represent the balance values, respectively. Start the algorithm with the following flow: $f(1,3) = 1, f(3,4) = 3, f(3,5) = 2, f(2,3) = 4, f(5,2) = 8$.
    (b) After computing the optimal flow f specify a potential function $\pi(v)$, $v \in V(G)$, on the vertices, such that the reduced costs $c_\pi(e)$ fulfill the inequalities $c_\pi(e) \geq 0$ for all $e \in E(G_f)$.

36. Solve the minimum cost flow problem for the network represented in Figure 1 by means of the successive-shortests-path-algorithm.

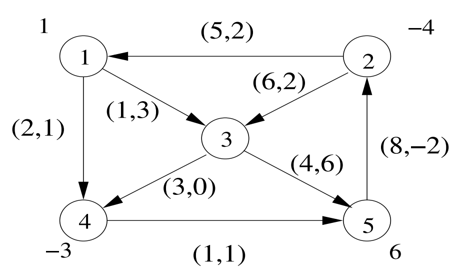

Figure 1: Input for the task 35 und 36