

Edmonds' Blossom Algorithm (EBA) for the Min Weight Perfect Matching Problem (MWPNP)

Input: $G=(V,E)$ undirected graph, $C:E \rightarrow \mathbb{R}$

Output: p.Matching Π with min. weight or the message "A p.M in G ".

(1) Initialize, $x_e := 0 \forall e \in E(G)$

$$z_{\{u,v\}} := \frac{1}{2} \min \{c_e : e \in \delta(v)\}, \forall v \in V(G)$$

$$z_A := 0, \forall A \in \mathcal{A} \quad |A| > 1$$

$$OPT = 0 \quad M_z := \emptyset \quad G_z := G$$

(2) While M_z is not perfect in G_z and $OPT < \infty$ do

(2a) Construct $G_z^=$

(2b) Apply EBA to compute a max cardinality matching M_z in G_z and the Edmonds-Gallai-Decomposition of G_z given by $D(G_z^=)$, $A(G_z^=)$, $C(G_z^=)$

(2c) If M_z is not perfect in G_z

Compute ϵ ; Set $OPT = \epsilon$;

If $\epsilon < +\infty$ then update z

end while

(3) If $OPT = \infty$ output "A p.M. in G "

else extend p.M. M_z in G_z into a p.M. Π in G

Output Π and $C(\Pi) := \sum_{e \in M} c_e$

Compute ϵ : $\epsilon = \min \{\epsilon_1, \epsilon_2, \epsilon_3\}$

$$\epsilon_1 = \min \{z_A : A \in \mathcal{A}, |A| > 1, A = B_v, v \in A(G_z^=)\}$$

$$\epsilon_2 = \min \{slack((x,y)) : x \in B_v, v \in A(G_z^=), y \in B_w, w \in C(G_z^=)\}$$

$$\epsilon_3 = \frac{1}{2} \min \{slack((x,y)) : x \in B_v, y \in B_w, v \in C_1, w \in C_2, C_1 \neq C_2, C_1, C_2 \text{ are conn. comp of } D(G_z^=)\}$$

Update z

$\forall A \in \mathcal{A}$ s.th. $A = \bigcup_{v \in C} B_v$ for some conn. comp. C of $D(G_z^=)$

set $z_A := z_A + \epsilon$

$\forall A \in \mathcal{A}$ s.th. $A = B_v$ for some $v \in A(G_z^=)$ set $z_A := z_A - \epsilon$