Combinatorial Optimization 2
Summer term 2019
Third work sheet

17. Given a graph G and a set $T \subseteq V(G)$, describe a polynomial time algorithm which finds a T-Join in G or decides that none exists. Can you give a linear time algorithm for this problem?

18. Consider a graph $G = (V, E)$ with infinite capacities on the edges, i.e. $u(e) = \infty$, $\forall e \in E$), with weights $c\colon E \to \mathbb{R}$ on the edges and with $b\colon V \to \mathbb{N}$, such that $\sum_{v \in V} b(v) = O(n)$, where $n = |V|$). Give a polynomial time algorithm which determines a b-matching with maximum weight in G.

    Hint: Transform the maximum weight b-matching problem to a maximum weight matching problem.

19. Show that the maximum weigh b-matching problem on a graph $G = (V, E)$ with capacities $u\colon E \to \mathbb{N} \cup \{\infty\}$ on the edges, weights $c\colon E \to \mathbb{R}$ on the edges and $b\colon V \to \mathbb{N}$, such that $b(v)$ is even $\forall v \in V(G)$, can be solved in strong polynomial time.

    Hint: Transform the particular maximum weight b-matching problem desribed above to an appropriate minimum cost flow problem.

20. Consider the graph G in Figure 1 and the set of vertices $T = \{1, 2, 5, 7\}$ in G. Determine a T-join with minimum weight in G. The numbers close to the edges specify the egde weigths.

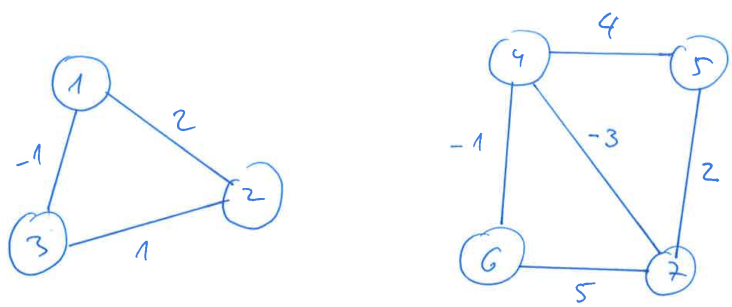

Figure 1: Input graph for Exercise 20

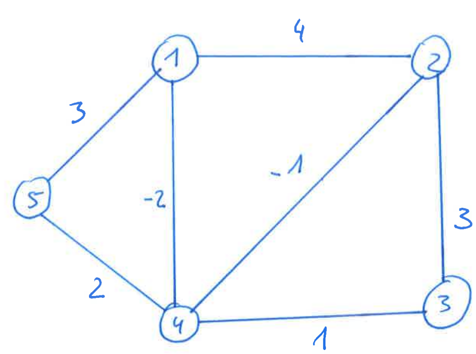

Figure 2: Inputgraph for Exercise 21 and Exercise 23

21. Consider the graph G in Figure 2. Apply the approach discussed in the lecture to determine a shortest $P_{s,t}$ path from s to t for any pair of vertices s and t, $s \neq t$, in G. The numbers close to the edges specify the edge weights.

22. Give an $O(|E(G)||V(G)|^3)$ algorithm to determine a cycle of minimum weight in an undirected, weighted graph G with conservative weights on the edges. Such an algorithm enables the computation of the girth of a graph in polynomial time. (The *girth* of an undirected graphen G is defined as the minimum number of edges of a cycle in G.)

23. Consider the graph G in Figure 2. Determine a cycle of minimum length in G by appling some systematic approach, e.g. the algorithm mentioned in Exercise 22. The numbers close to the edges specify the edge weights.

24. Consider an undirected graph G, a set $T \subseteq V(G)$ with even cardinality $|T|$ and a set of edges $F \subseteq E(G)$. Prove the following statemets:

    (a) F has a non-empty intersection with every T-join if and only if there existsa T-cut $C = \delta(X)$, $X \subset V$, such that $C \subseteq F$.

    (b) F has a non-empty intersection with every T-cut if and only if there exists a T-join J such that $J \subseteq F$.