Combinatorial Optimization 2
Summer Term 2019

fourth work sheet

25. Formulate a 2-factor approximation algorithm for the following problem. Given a digraph with nonnegative edge weights, find an acyclic subgraph of maximum weight.

    Note: No k-factor approximation algorithm for this problem is known for $k < 2$.

26. (a) Give a polynomial time algorithm for the minimum vertex cover problem in a bipartite graph.

    (b) Let $M \in \{0,1\}^{|V| \times |E|}$ be the incidence matrix of a graph $G = (V, E)$ with weights $c \colon E \to \mathbb{R}_+$ on the edges. Observe that $\min\{c^t x \colon M^t x \geq \mathbf{1}, \mathbf{0} \leq x \leq \mathbf{1}\}$ is a linear relaxation of the minimum weight vertex cover problem (with $\mathbf{0}$, $\mathbf{1}$ being the vectors of all zeroes and all ones in $\mathbb{R}^{|E|}$, respectively). Show that the minimization problem above has a half-integral optimal solution, i.e. an optimal solution $\hat{x}$ such that $\hat{x} \in \{0, 1/2, 1\}^{|V|}$.

    (c) Use the result of (b) to derive a 2-factor approximation algorithm for the minimum weight vertex cover problem.

27. Prove tha the simple greedy algorithm for the maximum cut problem discussed in the lecture is a 2 -factor approximation algorithm.