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Credit risk is the risk that the value of a portfolio changes due to unexpected changes in the credit quality of issuers or trading partners. This subsumes both losses due to **defaults** *and losses caused by* **changes in credit quality** *such as the downgrading of a counterparty in an internal or external rating system.*

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Examples of finance instruments affected by credit risk

- bond portfolios
- OTC ("over the counter") transactions
- trades with credit derivatives
- ...

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L is a r.v. and its distribution depends from the c.d.f. of $(X_1, \ldots, X_n, \lambda_1, \ldots, \lambda_n)^T$ ab.

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$S_i = 0$ corresponds to default.

Then we have $X_i = \begin{cases} 0 & S_i \neq 0 \\ 1 & S_i = 0 \end{cases}$

Models with latent variables (contd.)

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$S = (S_1, S_2, \ldots, S_n)^T$ is modelled by means of latent variables $Y = (Y_1, Y_2, \ldots, Y_n)^T$, e.g. Y_i could be the value of the assets of obligor i

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Let d_{ij}, $i = 1, 2, \ldots, n$, $j = 0, 1, \ldots, m+1$ be threshold values such that $d_{i,0} = -\infty$ und $d_{i,m+1} = \infty$ and $S_i = j \Longleftrightarrow Y_i \in (d_{i,j}, d_{i,j+1}]$.

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Let F_i be the distribution function of Y_i. The probability of default for obligor i is $p_i = F_i(d_{i,1})$.

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The probability that the fisrt k obligors default:

$$p_{1,2,\dots,k} := P(Y_1 \le d_{1,1}, Y_2 \le d_{2,1}, \dots, Y_k \le d_{k,1})$$

$$= C(F_1(d_{1,1}), F_2(d_{2,1}), \dots, F_k(d_{k,1}), 1, 1, \dots, 1) = C(p_1, p_2, \dots, p_k, 1, \dots, 1)$$

Thus the totalt defalut probability depends essentially on the copula C of $(Y_1, Y_2, \dots, Y_n)$.

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Notations:

$V_{A,i}(T)$: value of assets of firm i at time point T

$K_i := K_i(T)$: value of the debt of firm i at time point T

$V_{E,i}(T)$: value of equity of firm i at time point T

Assumption: future asset value is modelled by a geometric Brownian motion

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$$V_{A,i}(T) = V_{A,i}(t)\exp\left\{\left(\mu_{A,i} - \frac{\sigma_{A,i}^2}{2}\right)(T-t) + \sigma_{A,i}\left(W_i(T) - W_i(t)\right)\right\},$$

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where

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DD_i is called **distance-to-default**.