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(v) Let $g_X(t)$ be the pgf of X. Then $P(X=k) = \frac{1}{k!} g_X^{(k)}(0)$, where $g_X^{(k)}(t) = \frac{d^k g_X(t)}{dt^k}$.

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The loss function is then given by $L = \sum_{i=1}^n \bar{X}_i v_i L_0 \approx \sum_{i=1}^n X_i v_i L_0$, where $\bar{X}_i$ is the loss indicator and $(X_1, \ldots, X_n)$ has a PMD with factor vector $(Z_1, Z_2, \ldots, Z_m)$ as described above.

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$X_i|Z \sim Poi(\lambda_i(Z)), \forall i \Longrightarrow g_{X_i|Z}(t) = \exp\{\lambda_i(Z)(t-1)\}, \forall i \Longrightarrow$
$g_{N|Z}(t) = \prod_{i=1}^n g_{X_i|Z}(t) = \prod_{i=1}^n \exp\{\lambda_i(Z)(t-1)\} = \exp\{\mu(t-1)\}$,
with $\mu := \sum_{i=1}^n \lambda_i(Z) = \sum_{i=1}^n \left(\bar{\lambda}_i \sum_{j=1}^m a_{ij} Z_j\right)$.

The pgf of the loss distribution (contd.)

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Analogous computations as in the case of $g_N(t)$ yield:

$$g_L(t) = \prod_{j=1}^{m} \left(\frac{1-\delta_j}{1-\delta_j \Lambda_j(t)} \right)^{\alpha_j} \text{ wobei } \Lambda_j(t) = \frac{1}{\mu_j} \sum_{i=1}^{n} \bar{\lambda}_i a_{ij} t^{v_i}.$$

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$g_N^{(k)}(0) = \sum_{l=0}^{k-1} \binom{k-1}{l} g_N^{(k-1-l)}(0) \sum_{j=1}^{m} l! \alpha_j \delta_j^{l+1}$, where $k > 1$.