Risk theory and management in actuarial science
winter term 2020/21

third worksheet

19. (a) Compute the expectation $E(G_{\gamma,0,\beta})$ of the generalized Pareto distribution $G_{\gamma,0,\beta}$.
    (b) Consider a random variable X with distribution function F for which the approximation $\bar{F}_u(x) \approx \bar{G}_{\gamma,0,\beta(u)}(x)$ holds with $\gamma \notin \{0,1\}$. Show that this implies the approximation $\bar{F}_v(x) \approx \bar{G}_{\gamma,0,\beta(u)+\gamma(v-u)}(x)$, $\forall v \geq u$.
    (c) Use the results of (a) and (b) to show that for any fixed threshold $u > 0$ the mean excess function $e_X(v)$ is linear in v for $v \in [u, +\infty)$.

20. Use the peaks over threshhold (POT) method to analyse the tails of the data described in Exercise 18 (in the second worksheet).
    (a) Argue carefully upon your choice of the threshold parameter k based on the inspection of the plot of the empirical mean excess function (analogously to the case of the fire insurance example discussed in the lecture).
    (b) Maximize the log-likelihood function to obtain estimators for γ and β by using a solver of your choice. Consider the plot of the different values of the estimator $\hat{\gamma}$ of γ in dependence of the threshold parameter k to justify your choice for a suitable interval of values of k (cf. the fire insurance example from the lecture).
    (c) Compute estimators for $\mathrm{VaR}_{0.95}$ and $\mathrm{CVaR}_{0.95}$ for the whole interval of reasonable values of k determined in (b). Visualize the dependence of these estimators on k graphically and revise you choice for the interval of values of k, if appropriate.
    (d) Choose a value of k and visualize in one plot the empirical tail distribution and the tail distribution obtained by the POT method. Comment upon your results.

21. Let the random variables X_i, $i = 1, 2$, be such that $X_1 \sim Exp(\lambda)$ and $X_2 = t(X_1)$, where $Exp(\lambda)$ is the exponential distribution with parameter λ and $t\colon \mathbb{R} \to \mathbb{R}$, $t(x) = x^2$. Determine the coefficients of the lower and the upper tail dependence $\lambda_L(X_1, X_2)$, $\lambda_U(X_1, X_2)$, respectively, and conclude that X_1 and X_2 have both a lower and an upper tail dependence. Compute also the coefficient of the linear correlation $\rho_L(X_1, X_2)$, compare the three computed dependence measures and comment on your results.