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$$\hat{\bar{F}}(u+y) = \frac{N_u}{n}\left(1+\hat{\gamma}\frac{y}{\hat{\beta}}\right)^{-1/\hat{\gamma}} \text{ and } \hat{q}_p = u + \frac{\hat{\beta}}{\hat{\gamma}}\left(\left(\frac{n}{N_u}(1-p)\right)^{-\hat{\gamma}} - 1\right)$$

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For $\hat{\gamma} \notin \{0,1\}$ we get the following estimator for CVaR:

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The proof is done in two steps:

Estimation of VaR und CVaR by means of POT (contd.)

(1) Let X be a r.v. with $X \sim GPD_{\gamma,0,\beta}$ and $\gamma \notin \{0,1\}$. We show that

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The CVaR of the approximation $\tilde{F}$ is given as follows for $q_p > u$:

$$CVaR_p(\tilde{F}) = \hat{q}_p + \frac{\beta + \gamma(\hat{q}_p - u)}{1-\gamma}$$