Problem sheet 1
Jan 13th 2004

MT290 Complex variable

Ex. 1
Express the following complex numbers in the form $a + bi$:

$$\text{(a)}\;\; (4+3i)+(2+i), \quad \text{(b)}\;\; (4+3i)(2+i), \quad \text{(c)}\;\; \frac{1}{4+3i}, \quad \text{(d)}\;\; \frac{2+i}{4+3i}$$

Mark the position of each of these numbers on the Argand diagram and write down its conjugate.

Ex. 2
Find the modulus and argument of each of the complex numbers:
a) $1+i, \quad 1-i, \quad -1+i, \quad -1-i$
b) $\sqrt{3}+i, \quad -1+i\sqrt{3}, \quad \sqrt{3}-i, \quad 1-i\sqrt{3}$.
Mark the position of each of these complex numbers on the Argand diagram.

Ex. 3
Find the real and imaginary parts of $2e^{-i\pi/6}+\sqrt{2}e^{-3i\pi/4}$.

Ex. 4
Verify that for all complex numbers z, w we have:

$$\overline{zw}=\bar{z}\bar{w}, \qquad \overline{z+w}=\bar{z}+\bar{w}, \qquad \overline{z\overline{w}}=\bar{z}w, \qquad |z+w| \le |z|+|w|.$$

(for the last part you could use $|z+w|^2=(z+w)\overline{(z+w)}$, multiply this out, then show that the result is less than $(|z|+|w|)^2$).

Ex. 5
Let $z_1=r_1(\cos\theta_1+i\sin\theta_1), z_2=r_2(\cos\theta_2+i\sin\theta_2)$, where $r_1>0$, $r_2>0$, and let n be a positive integer. Express each of the following numbers in the form $r(\cos\theta+i\sin\theta)$ with $r>0$:

$$\text{(a)}\;\; z_1z_2, \quad \text{(b)}\;\; z_1^n, \quad \text{(c)}\;\; \frac{1}{z_1}, \quad \text{(d)}\;\; \frac{\sqrt{3}+i}{1-i}, \quad \text{(e)}\;\; \frac{i-\sqrt{3}}{-1-i}$$

(use your answers to q.2 for (d) and (e)).

Ex. 6
By writing z in modulus-argument form, solve the complex equations:
(a) $z^n=1$ for n a positive integer, (b) $z^6=-1$,
(c) $z^3=1$, (d) $z^2+z+1=0$, (e) $z^4+z^2+1=0$.
Remember:

$$\sin(\pi/4)=\cos(\pi/4)=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2},$$

$$\sin(\pi/6)=\cos(\pi/3)=\frac{1}{2}, \qquad \sin(\pi/3)=\cos(\pi/6)=\frac{\sqrt{3}}{2}.$$

Ex. 7
Let $f:\mathbb{C}\to\mathbb{C}$ be a function. Define: f is continuous at $c\in\mathbb{C}$.
Use the definition to show that f with $f(z)=z^3$ is continuous everywhere.

An electronic version of this problem sheet is available at
http://www.ma.rhul.ac.uk/~elsholtz/04mt290/lecture.html