Problem sheet 3
Jan 27th 2004

MT290 Complex variable

Ex. 1
Transform each of the following equations into a quadratic equation in e^z and hence find all the solutions for $z \in \mathbb{C}$ to

(a) $\cosh z = -1$; (b) $\sinh z = \frac{i\sqrt{3}}{2}$.

Ex. 2
Determine the principal value of the logarithm of each of the following complex numbers: (a) $-1 - i\sqrt{3}$, (b) $i - \sqrt{3}$, (c) $1 - i$, (d) $\dfrac{i - \sqrt{3}}{i - 1}$.
Hint: Some of the work has already been done on sheet 1 !

Ex. 3
Find the principal value, and all the other values, of the complex powers:
(a) $i^{\frac{2}{\pi}}$, (b) 1^{2i} (c) $(i - \sqrt{3})^i$, (d) $(1 - i)^{1+i}$.

Ex. 4
Let $f(x + iy) = u(x, y) + iv(x, y)$. Suppose that f is differentiable as a function of a complex variable, and that $v(x, y) = (u(x, y))^2$. Use the Cauchy-Riemann equations to show that

$$\frac{\partial u}{\partial x} = -4u^2 \frac{\partial u}{\partial x}$$

and thus deduce that $f(z)$ is constant. (Note: $1 + 4u^2 \neq 0$!)

Ex. 5
By writing $\tan z$ in terms of $w = e^{iz}$ write down the quadratic equation w must satisfy if

$$\tan z = a.$$

Hence find z if $\tan z = \sqrt{3} - 2i$. By considering the quadratic equation which w satisfies, and using the fact that $e^z = 0$ has no solution, show that $\tan z$ takes all values in $\mathbb{C}$ with two exceptions.