Problem sheet 4
Feb 3rd 2004

MT290 COMPLEX VARIABLE

Ex. 1
Sketch the following sets and investigate whether they are *open,* and/or *connected* (for (a)-(c) you will want to write $z = x + iy$):

(a) $\{z : \text{Im } z^2 > 1\}$; (*b*) $\{z : \text{Re } z^2 \leq 1\}$;
(c) $\{z : \text{Re } z^2 > 1, x > 1, y^2 < 4\}$; (*d*) $\{z : \arg z = \frac{\pi}{6}, 0 < |z| < 1\}$.

Ex. 2
Sketch and investigate the following contours to see whether or not they are *smooth, piecewise smooth, simple, closed*:
(a) $\phi(t) = t + i|t - 1|, \quad 0 \leq t \leq 2$
(b) $\phi(t) = 3 \sin t + 4i \cos t, \quad 0 \leq t \leq 2\pi$
(c) $\phi(t) = \sin t + i \sin 2t, \quad 0 \leq t \leq 2\pi$.

Ex. 3
Write down in the form $\phi(t),\ a \leq t \leq b$ the contour C consisting of a circle centre $1 + 0i$, radius 2 starting at $3 + 0i$ going in an anticlockwise direction. Hence evaluate the following two integrals:

$$\int_C z^2 \, dz, \qquad \int_C \frac{1}{z - 1} \, dz.$$

Ex. 4
Write down in the form $\phi(t),\ a \leq t \leq b$ the contour consisting of a circle centre i, radius 3 starting at $i - 3$ going in an anticlockwise direction. Hence evaluate the integral of $\bar{z}$ around this contour.

Ex. 5
Challenge Question. Let a simple closed smooth contour C be given by $\phi(t),\ a \leq t \leq b$ (anticlockwise direction as usual !). Let the area enclosed by C be A. Prove that the integral

$$\int_C \bar{z} \, dz$$

is pure imaginary. Show that the integral is $2iA$.