Problem sheet 10
March 16th, 2004

MT290 Complex variable

Ex. 1
Find the order of the zero for each of the following functions at $z = 0$: **a.** $z^{100} - z^{10}$; **b.** $e^z - 1 - z$; **c.** $\dfrac{\sin z - z}{z}$; **d.** $\dfrac{\cosh z - 1 - \frac{z^2}{2}}{z^2}$.

Ex. 2
Find the singularities of the following functions and determine whether or not they are poles:

$$\frac{z^2}{(z-2)^5(z-5)^7(z-10)^{100}}; \qquad \frac{e^z}{z^2} - \left(\frac{1}{z} + \frac{1}{2}\right)^2; \qquad \operatorname{cosec}(1/z).$$

Where the functions have poles, determine the order of the poles. Compare $\sin \frac{1}{z}$ and $\frac{1}{\sin z}$.

Ex. 3
Find the poles (with their order), and residues at the poles, for the following functions: **a)** $\dfrac{z}{z^2 - 3z + 2}$ **b)** $\dfrac{1}{e^z + 1}$ **c)** $\dfrac{\sin z}{z^3}$ **d)** $\dfrac{e^{z^2}}{(z^2+9)(z^2+25)}$.
Hint The function in b) has infinitely many poles.

Ex. 4
Find the poles of the following functions which have positive imaginary part, and calculate the residues at the poles.

$$\frac{z^2}{(z^2+1)(z^2+4)}; \qquad \frac{e^{iz}}{(z^2+1)(z^2+4)}.$$

Ex. 5
Evaluate the integral $\int_{|z|=1} \frac{e^{-z}}{z^2}\, dz$ (taken anticlockwise)
a) by means of Cauchy's integral formulae
b) by evaluating the residue
The same for $\int_{|z|=1} \frac{\sin(2z)}{z^2}\, dz$
$\int_{|z|=1} \frac{\sin(z)}{z^5}\, dz$
$\int_{|z|=1} \frac{\cosh z}{z^2}\, dz$
$\int_{|z|=1} \frac{\cosh z}{z^3}\, dz$.

If you have any questions on the course, previous problem sheets, exam or whatever, please write them up or send me an email (christian.elsholtz@rhul.ac.uk). In the last week I might come back to questions of general interest in the lecture.

Two further book recommendations:
Ian Stewart, David Tall Complex Analysis (more proofs, less examples than in our course).
Erwin Kreyszig: Advanced Engineering Mathematics, Chapters 12-15. (This book might also be useful for other courses). As the title suggests, the emphasis is on examples and problems, the theory is briefly summarized.