ROOM CHANGE: Wednesday 12 January, 10.00 from Lec B to Q170 (Queen's Building)

Problem sheet 1
Jan 11th 2005

MT290 Complex variable

Ex. 1
Express the following complex numbers in the form $a+bi$:

$$\text{(a)}\ (4+3i)+(2+i), \quad \text{(b)}\ (4+3i)(2+i), \quad \text{(c)}\ \frac{1}{4+3i}, \quad \text{(d)}\ \frac{2+i}{4+3i}$$

Mark the position of each of these numbers on the Argand diagram and write down its conjugate.

Ex. 2
Find the modulus and argument of each of the complex numbers:
a) $1+i, \quad 1-i, \quad -1+i, \quad -1-i$
b) $\sqrt{3}+i, \quad -1+i\sqrt{3}, \quad \sqrt{3}-i, \quad 1-i\sqrt{3}$.
Mark the position of each of these complex numbers on the Argand diagram.

Ex. 3
Find the real and imaginary parts of $2e^{-i\pi/6}+\sqrt{2}e^{-3i\pi/4}$.

Ex. 4
Verify that for all complex numbers z, w we have:

$$\overline{zw}=\bar{z}\bar{w}, \qquad \overline{z+w}=\bar{z}+\bar{w}, \qquad \overline{z\bar{w}}=\bar{z}w, \qquad |z+w|\le|z|+|w|.$$

(for the last part you could use $|z+w|^2=(z+w)\overline{(z+w)}$, multiply this out, then show that the result is less than $(|z|+|w|)^2$).

Ex. 5
Let $z_1=r_1(\cos\theta_1+i\sin\theta_1), z_2=r_2(\cos\theta_2+i\sin\theta_2)$, where $r_1>0$, $r_2>0$, and let n be a positive integer. Express each of the following numbers in the form $r(\cos\theta+i\sin\theta)$ with $r>0$:

$$\text{(a)}\ z_1z_2, \quad \text{(b)}\ z_1^n, \quad \text{(c)}\ \frac{1}{z_1}, \quad \text{(d)}\ \frac{\sqrt{3}+i}{1-i}, \quad \text{(e)}\ \frac{i-\sqrt{3}}{-1-i}$$

(use your answers to q.2 for (d) and (e)).

Ex. 6
By writing z in modulus-argument form, solve the complex equations:
(a) $z^n=1$ for n a positive integer, (b) $z^6=-1$,
(c) $z^3=1$, (d) $z^2+z+1=0$, (e) $z^4+z^2+1=0$.
Remember:

$$\sin(\pi/4)=\cos(\pi/4)=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2},$$

$$\sin(\pi/6)=\cos(\pi/3)=\frac{1}{2}, \qquad \sin(\pi/3)=\cos(\pi/6)=\frac{\sqrt{3}}{2}.$$

Ex. 7
Let $f:\mathbb{C}\to\mathbb{C}$ be a function. Define: f is continuous at $c\in\mathbb{C}$.
Use the definition to show that f with $f(z)=z^3$ is continuous everywhere.

- Possibly there will be a room change necessary during the term!
- **A one-time ROOM CHANGE due to First Year exams: Wednesday 12 January, 10.00 from Lec B to Q170 (Queen's Building)**
- Revise complex numbers from first year courses. Also revise some Calculus.
- An electronic version of the problem sheets will be (possibly with some delay) available at
  http://www.ma.rhul.ac.uk/~elsholtz/0405mt290/lecture.html
- Recommended text books are:
  Complex Analysis, J M Howie (Springer 2003). Library Ref. (515.24 How)
  Murray R. Spiegel: Theory and problems of complex variables. (510.76 Spi)
  Donald W. Trim: Introduction to Complex analysis and its applications. (515.24 Tri)
  Advanced Engineering Mathematics, 8th ed. E. Kreyszig (Wiley 1999). Library Ref. 510.245 KRE
  Note: Some books are on the restricted book shelf (behind the loan desk on the ground floor).
- Here is some good and some bad news: The good news is that there will be workshops especially for this course. The bad news is that traditionally this course is considered difficult so that the department thought this course most urgently needs backup by the workshops.

  However, workshops only fulfill their purpose if actually you are coming! There will be 3 workshops, but in case that the attendance of workshops drops rapidly, perhaps the number is reduced to 2.

  Especially if you think the problem sheets are too hard you should turn up at the workshops to get help!

  The workshops are: Thursday 3pm (held by myself)
  Friday 12 and 1pm (held by Dr Watt).
  If you are primarily interested in help on the harder questions please, if possible at all, come to the Friday 1pm workshop. I hope this gives a chance to everybody else to ask all other questions in the two first workshops.

  If you have questions on any other courses, of course you can also ask in the workshops but there will be in addition a list of colleagues who also are happy to answer questions on various courses. (Watch out at the notice board soon!)