Problem sheet 5
Feb 8th 2005

MT290 Complex variable

Ex. 1
Write down in the form $\phi(t),\ a \le t \le b$ the contour C consisting of a circle with centre $1+0i$, radius 2, starting at $3+0i$ going in an anticlockwise direction. Hence evaluate the following two integrals:

$$\int_C z^2\ dz, \qquad \int_C \frac{1}{z-1}\ dz.$$

Ex. 2
Write down in the form $\phi(t),\ a \le t \le b$ the contour consisting of a circle with centre i, radius 3, starting at $i-3$ going in an anticlockwise direction. Hence evaluate the integral of $\bar{z}$ around this contour.

Ex. 3
Let C be the triangular contour joining $0, 1, 1+i$ taken in the anticlockwise direction. Sketch C and write down the equations of each of the three lines making up C. Hence evaluate

$$\int_C \mathrm{Re}\, z\ dz.$$

Ex. 4
Let γ be the circle with centre 0 and radius r taken anticlockwise. Evaluate (for all r), using the definition of a contour integral,

$$\int_\gamma z^n\ dz,$$

where $n \in \mathbb{Z}$.

Ex. 5
Challenge Question. Let a simple closed smooth contour C be given by $\phi(t),\ a \le t \le b$ (anticlockwise direction as usual !). Let the area enclosed by C be A. Prove that the integral

$$\int_C \bar{z}\ dz$$

is pure imaginary. Show that the integral is $2iA$.

Note: Since nobody is coming to the workshops on Friday 1pm it is herewith cancelled. We will try to offer another workshop on Thursdays, possibly 11am, subject to availability of rooms; but anyway it only makes sense if some students are coming! A small number of students is coming on Friday 12pm. Could these please contact me or Dr. Watt: can you also come on Thursdays, 11am or 3pm? This week there will be workshops Thursday 3pm and Friday 12pm.