```
(* The purpose of the notebook is to explain a visual proof of the
   Fundamentaltheorem of algebra. We study the image of a small circle
    (which is a circle about a_0) and the image of a large circle,
  which is an n-fold loop about the origin. Blowing up the origanal
    circle smoothly has a continuous image wich must meet the origin n
    times. This gives n roots. The polynomial below is rather a random one.)
```

```
z[t_, r_] = r Exp[ I t]
```

$$e^{i\,t}\,r$$

```
(* this defines the complex variable in argument modulus form*)
```

```
f[z_] = 1 + I + z (2 + I) + z^2 (1 - I) + z^7
```

$$(1+i) + (2+i)\,z + (1-i)\,z^2 + z^7$$

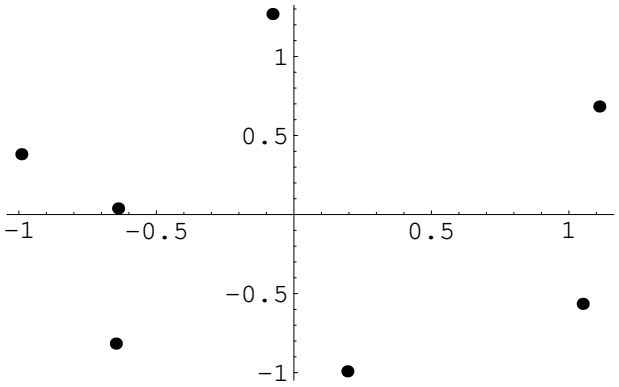
```
Table[N[Root[f[z], k]], {k, 1, 7}]
```

```
{-0.991073 + 0.381558 i,
 -0.647409 - 0.816128 i,
 -0.639174 + 0.0388261 i,
 -0.0777882 + 1.26827 i, 0.1945 - 0.991401 i,
 1.05031 - 0.564596 i, 1.11064 + 0.683466 i}
```

```
Table[{Re[q = N[Root[f[z], k]]], Im[q]},
 {k, 1, 7}]
```

```
{{-0.991073, 0.381558},
 {-0.647409, -0.816128},
 {-0.639174, 0.0388261},
 {-0.0777882, 1.26827},
 {0.1945, -0.991401},
 {1.05031, -0.564596}, {1.11064, 0.683466}}
```

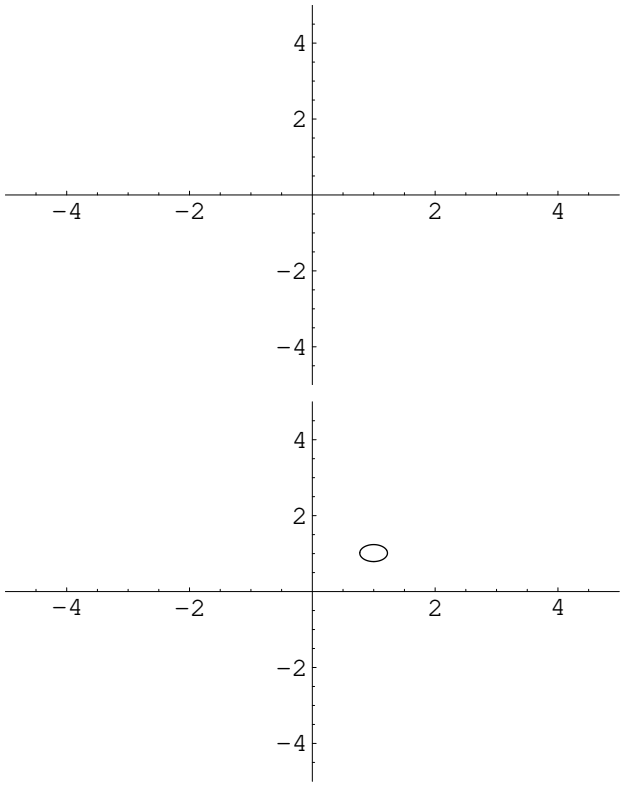
```
ListPlot[%, PlotStyle → PointSize[0.02]]
```

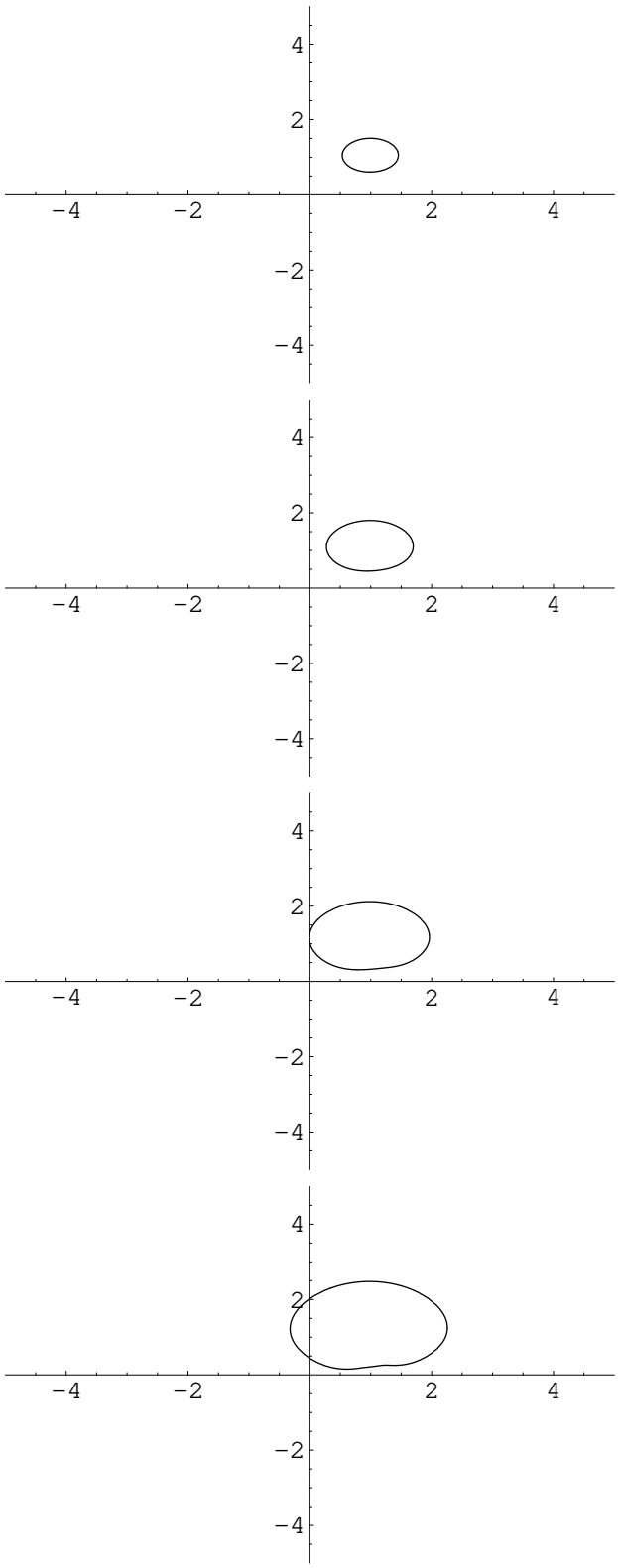


- Graphics -

```
Table[ParametricPlot[
  {Re[f[z[t, r]]], Im[f[z[t, r]]]},
  {t, 0, 2 Pi}, PlotRange →
   {{-5, 5}, {-5, 5}}], {r, 0, 1.5, 0.1}]
```

```
(*The range of t is a full circle, the readius r is increasing. Below
  you see theIMAGE of the circles when put into the polynomial f. *)
```

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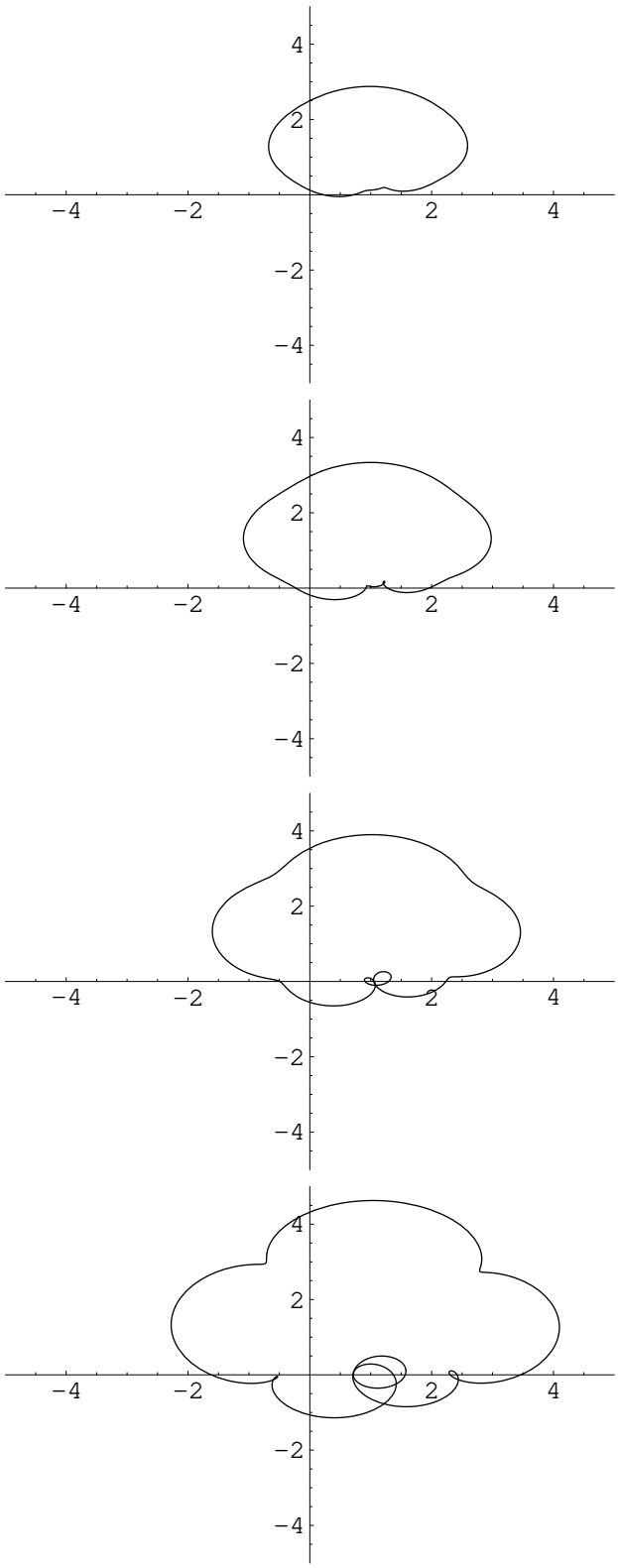
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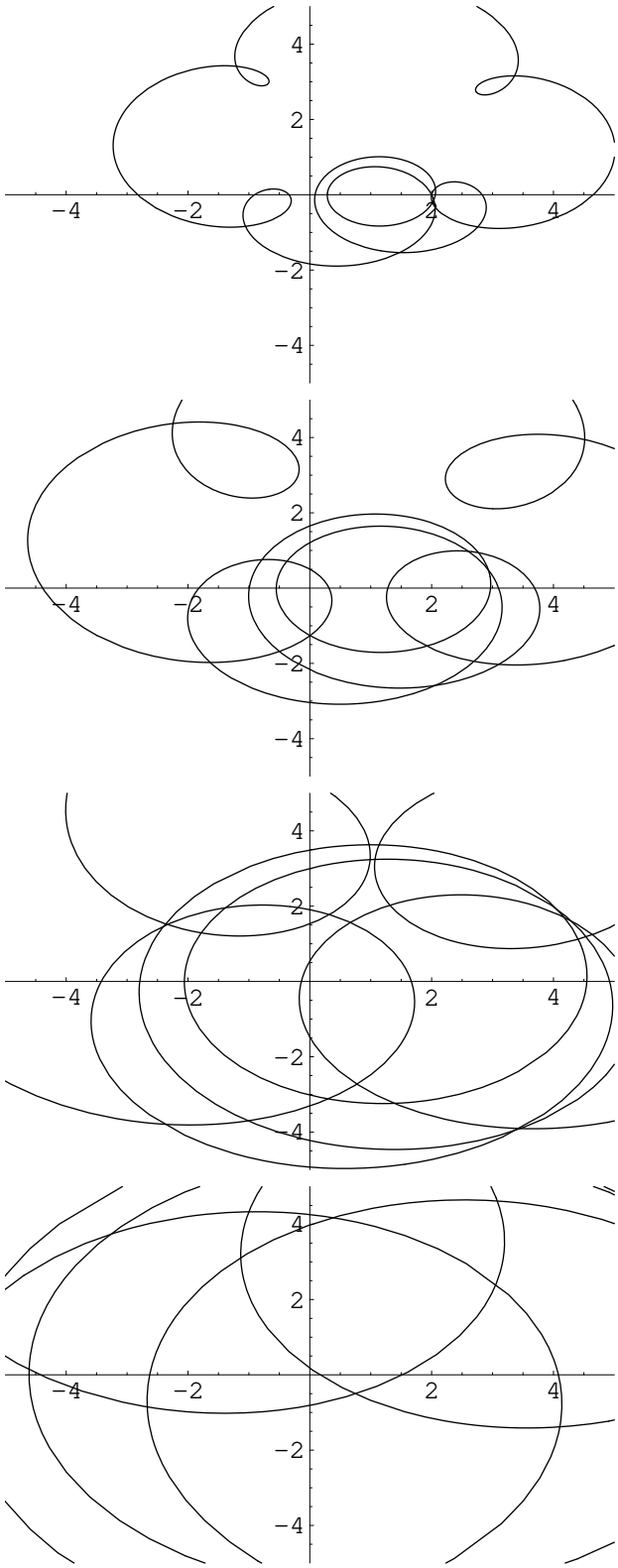
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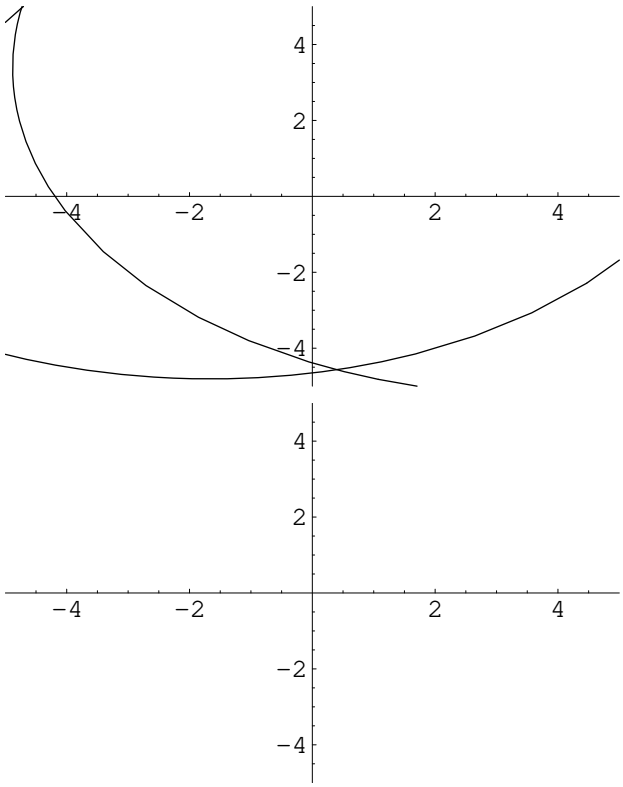

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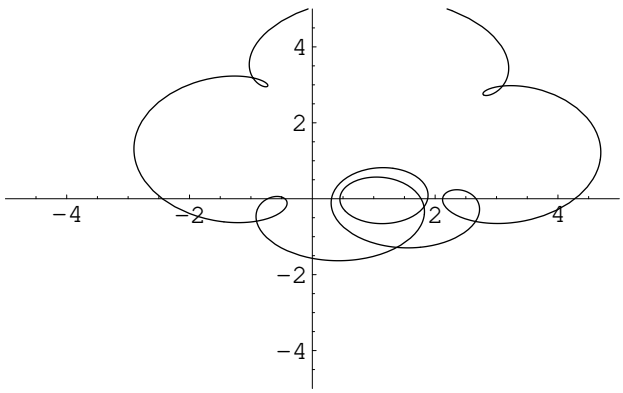
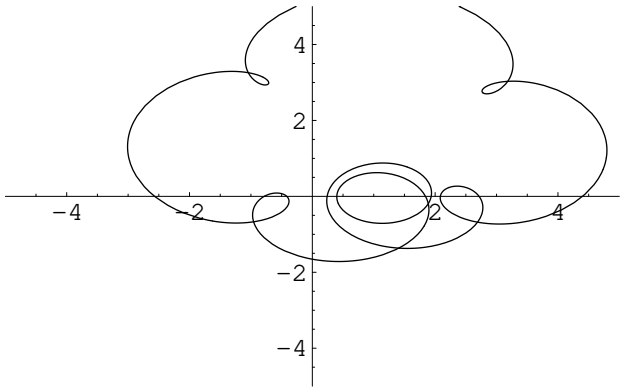
```
(*looking more closely at r about 1*)

Table[ParametricPlot[
  {Re[f[z[t, r]]], Im[f[z[t, r]]]},
  {t, 0, 2 Pi}, PlotRange →
   {{-5, 5}, {-5, 5}}],
 {r, 0.97, 1.1 , 0.01}]
```

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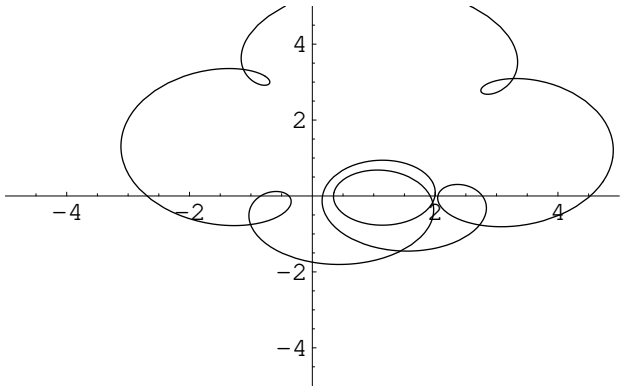
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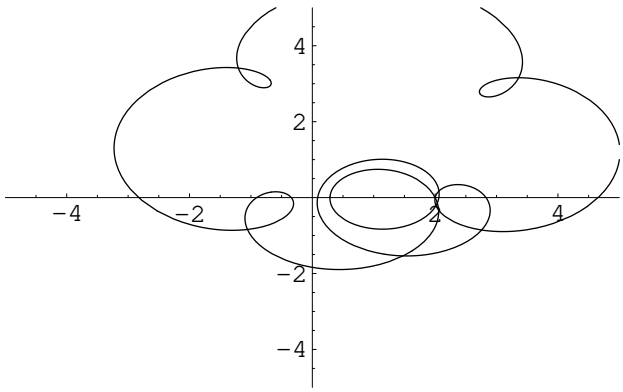
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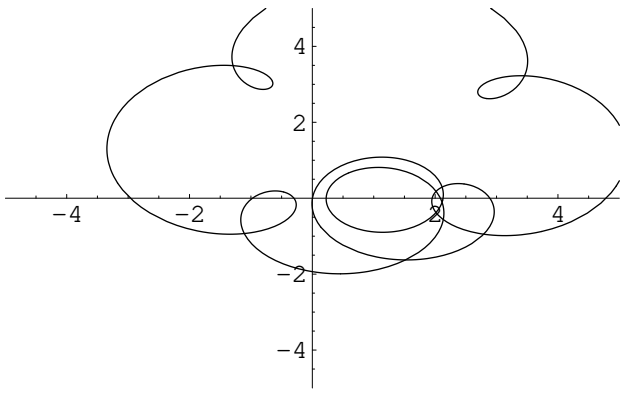
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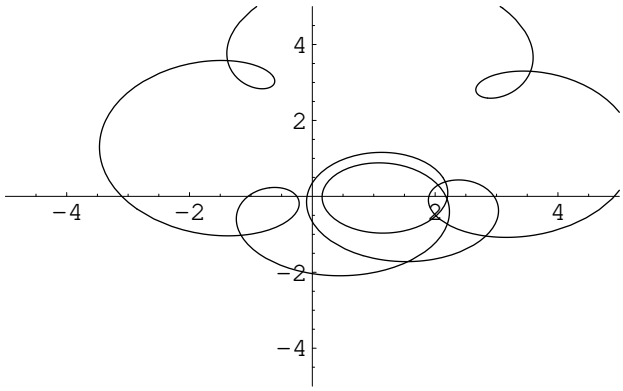
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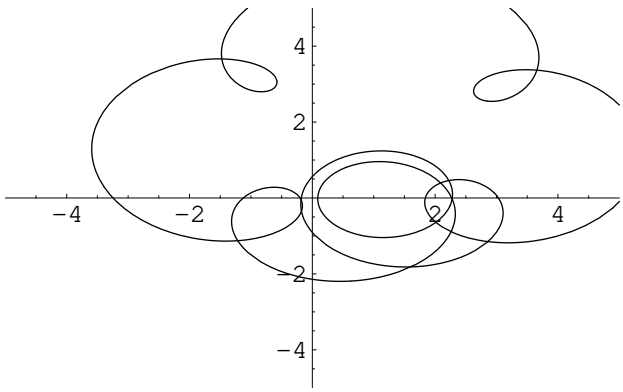
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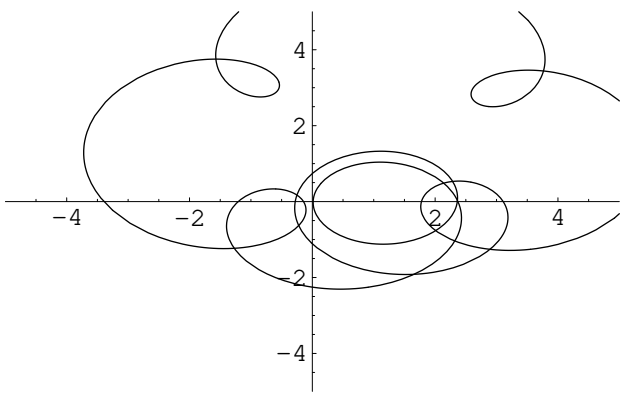
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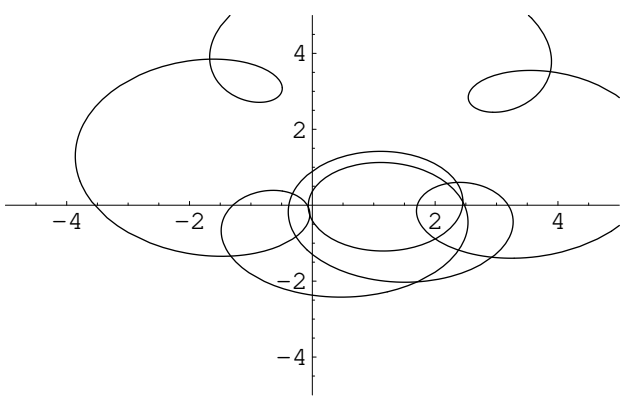
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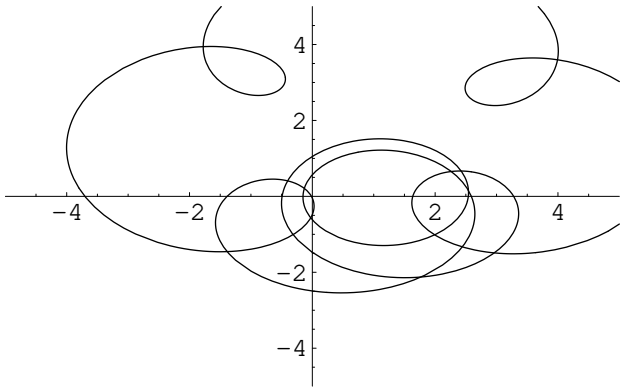
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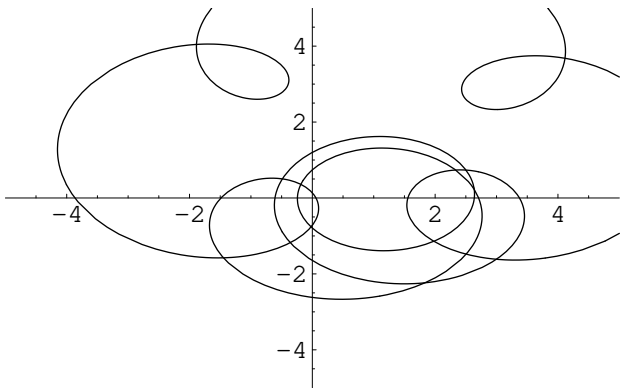
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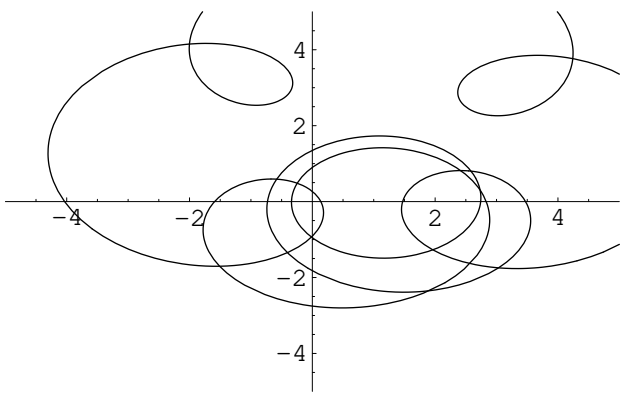
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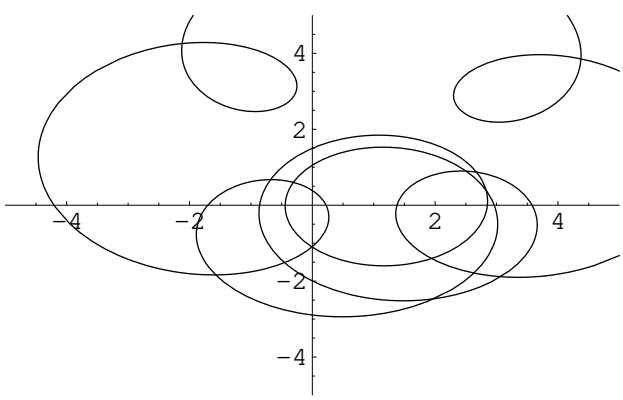
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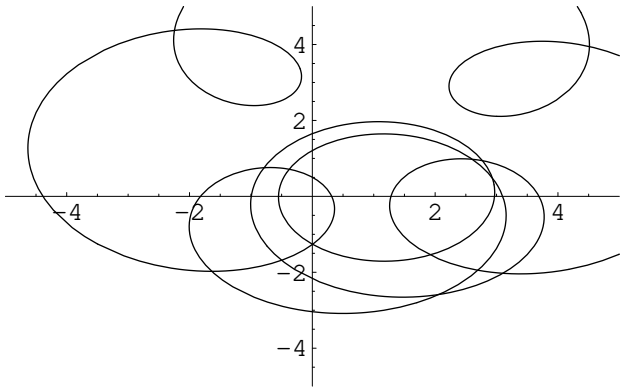

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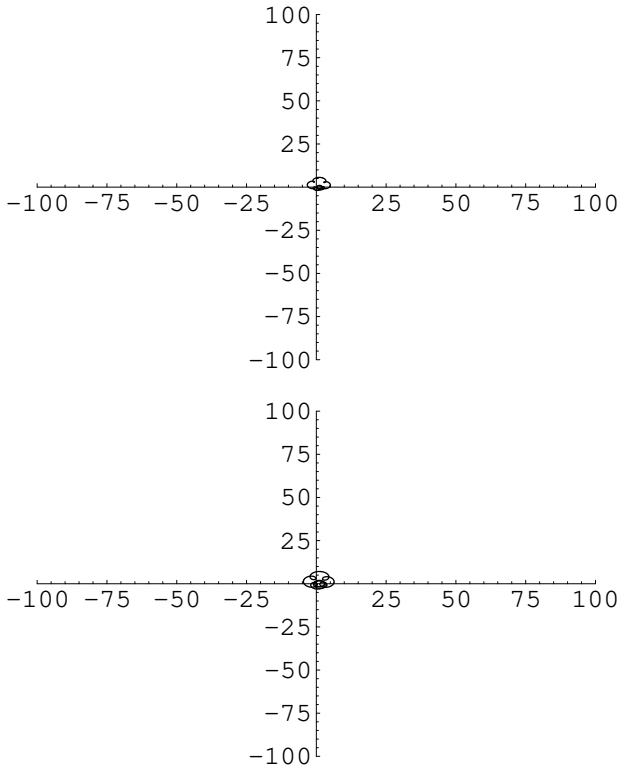
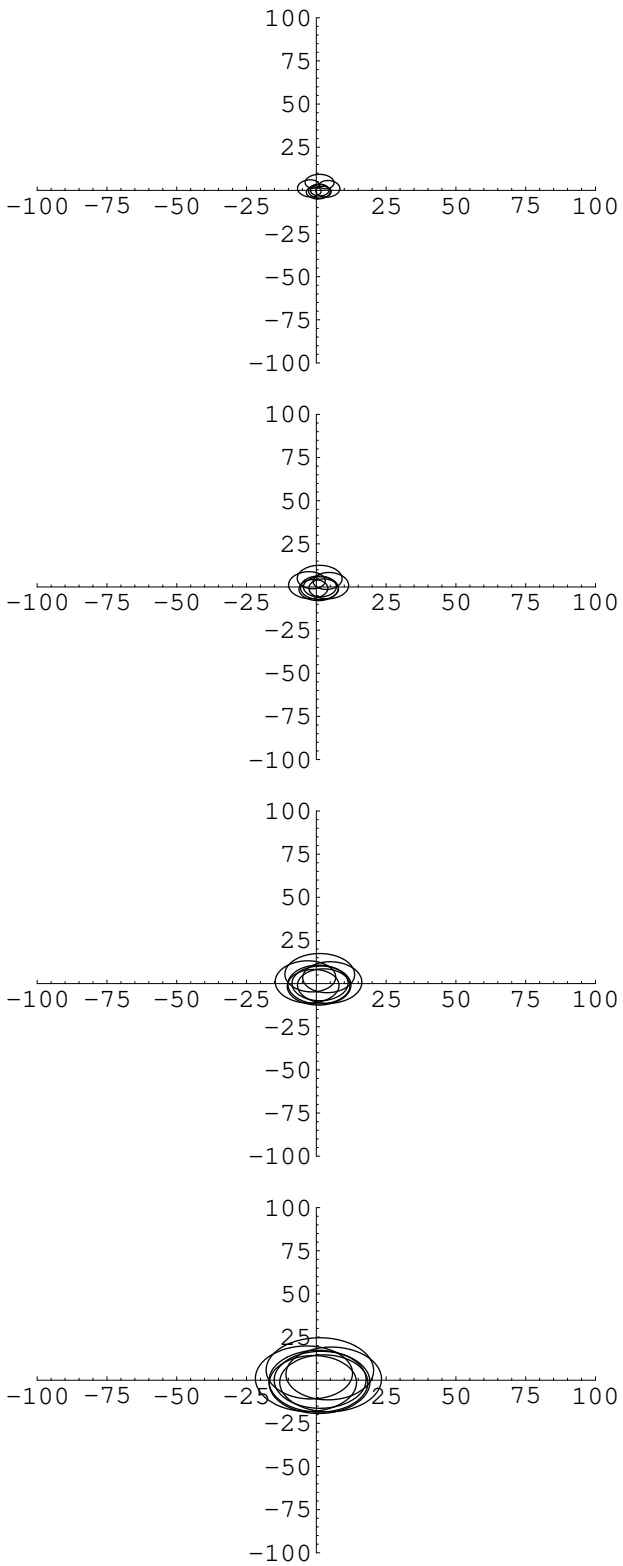
(* below are larger radii r *)

```
Table[ParametricPlot[
  {Re[f[z[t, r]]], Im[f[z[t, r]]]},
  {t, 0, 2 Pi}, PlotRange →
   {{-100, 100}, {-100, 100}}],
 {r, 1, 1.8 , 0.1}]
```

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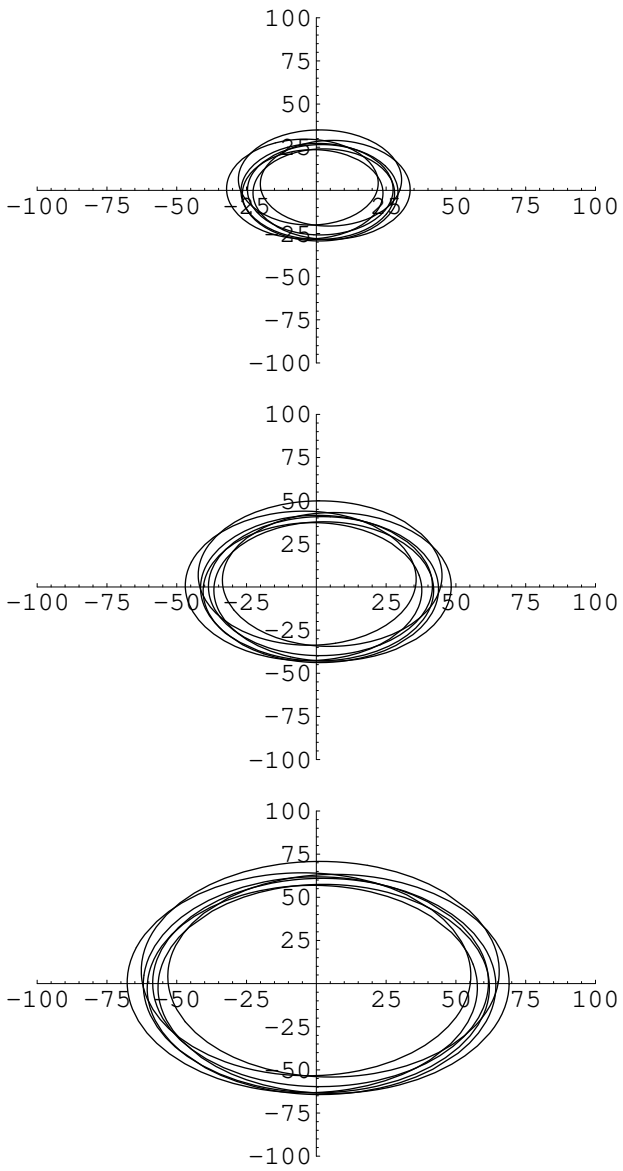

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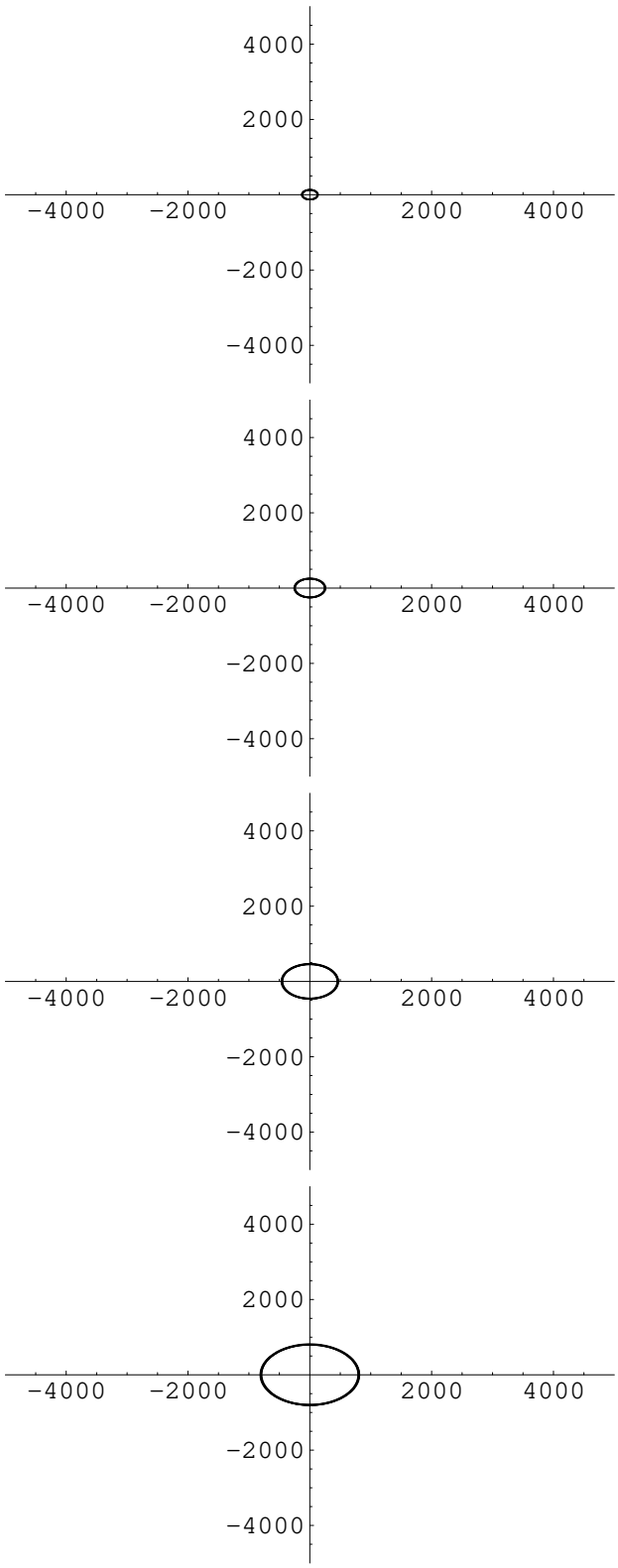
```
(* still larger radii r *)

Table[ParametricPlot[
  {Re[f[z[t, r]]], Im[f[z[t, r]]]},
  {t, 0, 2 Pi}, PlotRange →
   {{-5000, 5000}, {-5000, 5000}}],
 {r, 2, 3.4 , 0.2}]
```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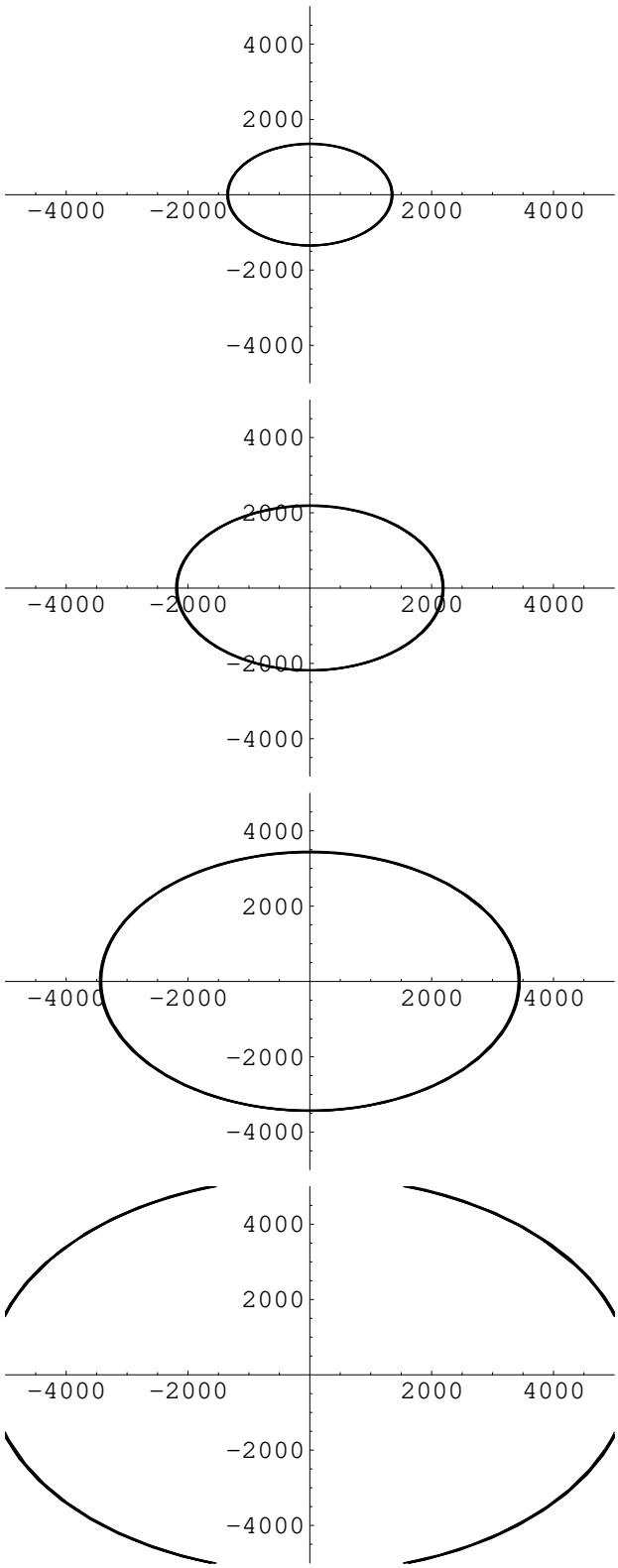

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```
(* the images are n fold loops, circle like.
   It looks like an ellipse. Mathematica has
  (automatically) chosen a different scale for the axes*)
```