Problem sheet 6 2005, Feb. 17th

Ex. 1

There exists a perfect [23, 12, 7] code. Determine the number of codewords of weight $0, 1, 2, 3, \ldots, 23$ (use the code is perfect!) From this determine the probability p_{correct} of correct decoding a word of length 23.

Ex. 2

Prove: Suppose C is a linear [n, k]-code having generator matrix G. Then a vector $\vec{v} \in V(n, q)$ belongs to $C^{\perp} \Leftrightarrow \vec{v}G^{\top} = 0$, where G^{\top} denotes the transpose of G.

Ex. 3

Determine the dual codes C_1^{\perp} and C_2^{\perp} for $C_1 = \{000, 111\}$ and $C_2 = \{00000, 11111\}$.

Ex. 4

Prove that if E_n is the binary even weight code of length n, then E_n^{\perp} is the repetition code of lengt n. Give the generator matrices and the parity check matrices for the two codes.

Ex. 5

Let C be the ternary linear code with generator matrix

$$\left[\begin{array}{rrrr} 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \end{array}\right].$$

a) Find a generator matrix G for C in standard form

b) Find a parity check matrix H for C in standard form.