

Problem sheet 7
2005, Feb. 24th

MT361 ERROR CORRECTING CODES

Ex. 1

(Not to be handed in) Work through pages 74-78 of Raymond Hill's book. In particular watch out for the advantages of incomplete decoding.

Ex. 2

Show that the decimal code

$$\left\{ (x_1, x_2, \dots, x_{10}) \in \mathbb{Z}_{10}^{10} \mid \sum_{i=1}^{10} x_i \equiv 0 \pmod{10}, \sum_{i=1}^{10} ix_i \equiv 0 \pmod{10} \right\}$$

is not a single-error-correcting code. (Here \mathbb{Z}_{10} denotes the set of integers mod 10.)

Ex. 3

(see problem sheet 6) Let C be the ternary linear code with generator matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}.$$

- Find a generator matrix for C in standard form
- Find a parity check matrix for C in standard form.
- Use syndrome decoding to decode the received vectors 2121, 1201 and 2222.

Ex. 4

Suppose a certain binary channel accepts word of length 7 and that the only kind of error vector ever observed is one of the eight vectors

0000000, 0000001, 0000011, 0000111, 0001111, 0011111, 0111111, 1111111.

Design a binary linear $[7, k]$ -code which will correct all such errors with as large a rate as possible.

Ex. 5

To determine the minimum distance of a general code one needs $\binom{M}{2}$ comparisons. Show that for a linear code the minimum distance is given by the minimum weight of any non-zero codeword.

Further show that C has distance d if and only if any $d - 1$ rows of a parity check matrix H are linearly independent but there exist d rows which are dependent.

Ex. 6

Let C be the code generated by

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

A new code C_2 is formed by adding a parity check.

- a) Write down the generator matrix for C_2 in standard form.
- b) Determine the parameters $[n, k, d]$ and M .
- c) Write down its parity check matrix for C_2 .
- d) Show that C_2 can be used to simultaneously correct one error and detect two errors.
- e) Make a lookup table to enable syndrome decoding to use it in the way suggested in d .
- f) Decode the received vectors 0110110, 1111010 and 1110111.

Do not write down the standard array or the codewords.