Exercise Sheet 5

MT454 Combinatorics

Note: On Friday 12th November I give a lecture in Exeter, i.e. the MT454 is likely to be cancelled.
Are alternative times like Monday 3pm or Monday 4pm (on a Monday to be determined) possible for you?

1. (a) Calculate the Möbius function of the following poset:

   (b) By generalising the above example, show that for all positive integers n there exists a poset P and $x, y \in P$ such that $\mu(x, y) = n$ (where μ is the Möbius function of P).

2. Recall that the Fibonacci Numbers $F_0, F_1, F_2, \ldots$ are defined by $F_0 = F_1 = 1$, and $F_{n+2} = F_{n+1} + F_n$ for all non-negative integers n. Use the techniques of Theorem 4.1 to show that
$$F_n = \left(\frac{\sqrt{5}+1}{2\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{\sqrt{5}-1}{2\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n.$$

3. (a) There are n seats arranged in a line. Show that the number of ways of choosing a subset of these seats, with no two chosen positions consecutive, is F_{n+1}.

   (b) If the n seats are now arranged in a circle, show that the number of choices is $F_n + F_{n-2}$ for $n \geq 2$.

4. By using long division, find the first four terms in the power series for
$$\frac{1+4x}{1+5x+x^2}.$$

5. Use partial fractions to simplify
$$\frac{1+3x}{1-3x^2+2x^3}.$$