Exercise Sheet 7

MT454 Combinatorics

1. Find the values of $p(n)$ where $1 \leq n \leq 7$ by writing down all the appropriate partitions.

2. Define $p_k(n)$ to be the number of partitions of the integer n with exactly k parts. (So $p_2(4) = 2$, as $2 + 2$ and $3 + 1$ are the two partitions of 4 with 2 parts.) Show that

$$p_k(n) = p_k(n-k) + p_{k-1}(n-k) + \cdots + p_0(n-k),$$

and use this equality to calculate $p(8)$, (where p is the standard partition function).

3. Write down the generating functions for the sequences whose nth terms are:

   (a) the number of partitions of n into parts equal to 3 or 5.

   (b) the number of partitions of n into parts equal to 5, 10 or 20.

4. By multiplying the appropriate power series, find the coefficient of x^9 in

$$\frac{1}{(1-x)(1-x^2)(1-x^3)}.$$

What is the interpretation of your result in terms of the number of partitions of a certain kind?