Note: If you found it difficult to do problem sheet 2 you should definitely attend the lectures regularly. Moreover there is no guarantee that there will be handouts.

Ex. 1
Revise your knowledge on finite fields and vector spaces (MT182/282).

Ex. 2
Suppose Bob wants to transmit single bit messages over a binary symmetric channel. He uses a repetition code of length 10. List the possibilities to use the redundant bits for schemes that detect and/or correct errors. (Up to how many errors can be detected? Corrected? Mixed forms of corrections and detections?)

Ex. 3
Revise your knowledge on geometric series, in particular:

$$\sum_{i=0}^{\infty} q^i = 1 + q + q^2 + q^3 + \ldots = \frac{1}{1-q}, \text{ if } |q| < 1.$$

(Exam question 1999)
Define the term "binary symmetric channel with cross-over probability p". Such a channel is used to send a message using one of two possible schemes as follows:-
A. in single bits using a 3-repetition code, correcting any detected errors;
B. in two bit words encoded with the addition of a check-sum bit, and requiring retransmission with any detected errors.
For each of these schemes,

1. write down the complete code;
2. find the overall probability of accepting an error;
3. find the expected number of bits that have to be transmitted per message bit.

Calculate the above quantities for $p = 0.01$ and $p = 0.1$.
Explain the relative merits of these methods under various circumstances.

Ex. 4

a) Alice and Bob play the following game: Alice thinks of a number $n \in \{1, 2, \ldots, 1\ 000\ 000\}$. Bob is allowed to ask questions and Alice will answer them (truthfully) with yes or no, only. Which number of questions guarantees that Bob can find the correct answer? Prove upper and lower bounds on the number of questions for the optimal strategy, and describe the strategy.

b) Now Alice is allowed to lie, but at most once. As before, how many questions does Bob have to ask, at least, and how many questions guarantee that he finds the correct answer? (In other words: Let a be the number of questions that the optimal procedure needs. Give lower and upper bounds on a.) Describe your procedure, describe the questions.