If you haven't done homework so far: Work through the homework and solutions of the previous weeks. If you want you can hand it in and get feedback. Please catch up with the material!

Ex. 1
A memoryless source emits only vowels, each with the following probabilities:

$$P(A) = 0.2, P(E) = 0.3, P(I) = P(O) = 0.2, P(U) = 0.1.$$

Estimate the number of typical outputs of length n. (Describe what you are doing: define what you mean by typical.)

Ex. 2
A memoryless source over the 26-letter alphabet has a vocabulary of about 10^n sequences of length n, for suifficiently large n. Estimate the entropy of the source. (Hint: The answer is simple and short.)

Ex. 3
Consider the infinite square lattice consisting of all integer-coordinated points of the plane and with nearest neighbours in the direction of the coordinate axes joined by an edge. A self avoiding walk of length n is a sequence of n edges starting from the origin, each pair of consecutive edges having a common point, and at no stage revisiting a point already visited. If $f(n)$ denotes the number of self-avoiding walks of length n, then $f(1) = 4, f(2) = 12$ and so on. Prove that

$$f(m+n) \leq f(m)f(n),$$

and hence deduce that

$$\lim_{n\to\infty} (f(n))^{\frac{1}{n}} = \inf_{n\geq 1} (f(n))^{\frac{1}{n}} = \theta$$

exists. Determine $f(3)$. Draw the situation for $n = 1, 2, 3$. Prove that $2 \leq \theta \leq 3$.

Ex. 4
With probability $\frac{1}{3}$, a source $\mathcal{S}$ emits a random string of zeros and ones; with probability $\frac{2}{3}$, it emits a random string of ones and twos. Show that the source is not ergodic.

Ex. 5
Find the entropy of the Markov source whose transition matrix is given by

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Ex. 6
Which of the Markov sources having transition matrices as shown are irreducible?

$$M_1 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}, \quad M_2 = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$M_3 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \end{pmatrix}, \quad M_4 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

To be returned in one week, before the lecture.
My web page contains a collection of related material.
http://www.ma.rhul.ac.uk/∼elsholtz/WWW/lectures/0506mt441/lecture.html