Ex. 1

(i) Draw the graphs or multigraphs G_1, G_2, G_3 for which the adjacency list (for G_1), adjacency matrix (for G_2) and incidence matrix (for G_3) are given below. Write down the adjacency matrices for G_1 and G_3.

For G_1:

a	b	c	d
b	a	a	a
c	c	b	c
d		d	

for G_2: $\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 & 0 \end{pmatrix}$ for G_3: $\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$

(ii) Write down the adjacency *list* for the first form of the Petersen graph given in Figure 1.2.2(a) and the adjacency matrix for the second form in Figure 1.2.2(b), using the labels shown used in the lecture.

(iii) Draw the graph G for which $V(G) = \mathbb{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $\{u, v\} \in E(G)$ if and only if u and v are distinct and coprime, (i.e. their greatest common divisor is 1).

Ex. 2

(i) What is the sum of the entries in the v-row (or v-column) of the *adjacency matrix* of a graph?

(ii) What is the sum of the entries in the v-row of the *incidence matrix* of a graph? And the sum of the entries in a column of this matrix?

(iii) Deduce the Handshaking Lemma for a graph from (ii) of this question.

Ex. 3

(i) Write down the valency numbers for the graphs $N_n, K_n, L_n, C_n, W_k, K_{r,s}$ defined in section 1.2, and for the Petersen Graph.

(ii) Determine which of the following sets of integers could be the valency numbers of a graph. If there is a graph, draw it. If not, try to find a multigraph with the same set of valency numbers.
(a) 1 1 2 3 (b) 1 1 2 3 3 (c) 2 2 4 4 4 (d) 1 2 2 3 (e) 0 1 2 3 4.

Ex. 4

(i) Draw all the non-isomorphic graphs for which $n = 1, 2$ or 3.

(ii) Find the 11 non-isomorphic graphs with $n = 4$.
Hint: Their sets of valency numbers are different.
Note: There are 34 non-isomorphic graphs with $n = 5$ and 156 with $n = 6$, and 1044 with $n = 7$ and 12346 with $n = 8$.

(iii) The example of section 1.4 (Figure 1.4.3) gives two non-isomorphic graphs with $n = 5, m = 5$ and the same set of valency numbers. Find the other two pairs of non-isomorphic graphs with $n = 5$ for which the sets of valency numbers are the same.
Hint: Try $m = 4$ and $m = 6$.