Ex. 1

i) Draw spanning trees for the graphs C_8, W_8, K_8 and the Petersen Graph.

ii) State with reasons whether the graphs $K_n, L_n, C_n, W_k \quad (n \geq 3, k \geq 3)$ and the Petersen Graph are
(a) Eulerian, (b) semi - Eulerian, (c) Hamiltonian, (d) semi - Hamiltonian.

(Note: In this course semi - Eulerian graphs have an Eulerian trail but not an Eulerian circuit and are thus not Eulerian. Similarly semi-Hamiltonian graphs are not Hamiltonian.

Ex. 2

i) Find spanning trees, and if possible Eulerian circuits or trails and Hamiltonian cycles or paths for the graphs G_1, G_2, G_3 shown on page 2 (where all the vertices are included see (ii) below), and so determine whether they are Eulerian, semi - Eulerian, Hamiltonian or semi - Hamiltonian.

ii) Let H_2 and H_3 be the multi-graphs obtained from G_2 and G_3 by removing the black vertices and joining up the edges incident with them, so that G_2 is a subdivision of H_2 etc., (see page 2). Which of the four properties listed in (i) are satisfied by H_2 and H_3, and how do the latter compare with G_2 and G_3 in this respect?

iii) Use Fleury's Algorithm to find an Eulerian circuit for the graph G_4 shown on page 2, starting at the vertex v_0, and an Eulerian trail for G_5. Give sufficient steps to show the method.

Ex. 3

i) Clearly no trees are Eulerian or Hamiltonian. Find all the trees which are semi - Eulerian or semi - Hamiltonian.

ii) For which values of r and s is $K_{r,s}$
(a) Eulerian, (b) semi - Eulerian, (c) Hamiltonian, (d) semi - Hamiltonian?

Ex. 4

i) The game of dominoes consists of all 28 pieces (i, j) with $0 \leq i \leq 6$ and $0 \leq j \leq 6$. One can put pieces consecutively if these pieces share the same number at the edge; example (1,5)(5,3). One can put pieces along lines or along columns). Show that it is possible to complete a game of dominoes by placing them in a single large cycle (i.e. the last piece fits together with the penultimate and with the first one). (Hint: Construct a multigraph for which the vertices are the different sets of dots (i.e 0 to 6) and the dominoes represent the edges.)

ii) How many additional bridges would the citizens of Königsberg have had to have built to ensure that there was an Eulerian circuit? Could they have solved this problem in another way?

iii) A very important application of Hamiltonian cycles is the Travelling Salesman Problem, where for a weighted graph the length of the cycle, i.e. the sum of the weights on its edges, is to be minimised. By using trial and error methods, find the shortest Hamiltonian cycle for the weighted complete graph G_6 shown on page 2.