**Ex. 1**

i) Use the Pigeonhole Principle to show that

a) given any $n+1$ different real numbers in $[0, 1)$, at least two of them satisfy $0 < x-y < \frac{1}{n}$;

b) given 5 points inside a unit square, at least two are less than $\frac{1}{\sqrt{2}}$ apart;

c) given any $n+1$ distinct integers from the set $M_{2n} = \{1, 2, \ldots, 2n\}$ then at least two of them satisfy $a \mid b$, i.e. $b$ is an integer multiple of $a$, or equivalently $a$ is a divisor of $b$.

ii) Suppose that each letter in a set $A$ is put into one of a set $B$ of pigeonholes. Prove by contradiction or otherwise that if $|A| > m|B|$, then at least one pigeonhole contains more than $m$ letters.

**Ex. 2**

i) Find relations on suitable sets $A$, which are

a) reflexive and transitive, but not symmetric;

b) reflexive and symmetric, but not transitive;

c) symmetric only.

ii) Explain what is wrong with the following argument, noting that in the definitions given there is no reason why $a, b, c$ should be distinct elements of $A$.

"If a relation is both symmetric and transitive it must also be reflexive, for if $a \sim b$ then from the symmetric law (ii) $b \sim a$, whence from the transitive law $a \sim b$ and $b \sim a$ giving $a \sim a$."

Hence or otherwise find a relation that is symmetric and transitive but not reflexive. How can such a relation be made into an equivalence relation on a suitable subset of $A$?

**Ex. 3**

i) Of a group of second year students, 24 are taking MT280, 20 are taking MT271, and 19 are taking MT261. Of these, 7 are taking MT280 and MT271, 5 are taking MT280 and MT261, and 4 are taking MT271 and MT261. What are the bounds on the number taking at least one of these units?
Hint: Assume that $t$ students are taking all three units and use the inclusion-exclusion principle (part i).

ii) If in the problem of part (i) only one student is taking all three units, use the inclusion-exclusion principle (part ii) to determine how many of the students are taking precisely one of the units.