

**Ex. 1**

- i) Consider the partitions of 10. As in section 2.2: (a) write down those partitions with at most 3 terms and match them with those with terms  $a_i \leq 3$ ;  
(b) match the partitions which have distinct odd terms with the self-conjugate partitions;  
(c) match the partitions which have distinct terms with those which have odd terms.
  
- ii) Show that there is only one natural number  $n$  for which there are no self-conjugate partitions.  
Hint: Consider the partitions of  $n$  which have distinct odd terms.

**Ex. 2**

- i) Following the method given in the lectures find the generating functions for the partitions of  $n$  which satisfy the following conditions. Leave the answers in the form of infinite products.
  - (a) The terms are all even. (Hint: Consider which values of  $t$  can occur in  $1 + x^t + x^{2t} + \dots$ )
  - (b) The terms are all distinct. (Hint: Consider which values of  $s$  can occur in  $\sum_s x^{st}$ .)

**Ex. 3**

- i) Verify Theorem 2.3.2 (i) for  $n = 3, r = 2$ .
- ii) Verify Theorem 2.3.2 (ii) for  $n = 4$ , and (iii) for  $n = 4, k = 3$ .
- iii) Find the values of  $S(n, r)$  for  $0 \leq n \leq 4, 0 \leq r \leq n$ .  
Add two further rows to the table of the Stirling numbers of the second kind.
- iv) How many ways are there of dividing  $M_4$  into disjoint non-empty subsets? What does this tell you about the equivalence relations on  $M_4$ ?

**Ex. 4**

- i) If there are 3 identical red cubes, 2 identical blue cubes and 1 green cube, in how many ways can all 6 be arranged in an ordered row? How many of these arrangements start and end with the same colour?
- ii) How many different signals can be sent by flying 3 identical red flags and 2 identical blue flags on a vertical rope, using some or all of the flags?  
Hint: First assume that  $r$  red flags and  $b$  blue ones are used and then add the results.
- iii) In how many ways can 4 people be given 16 identical biscuits if each person gets at least 2?
- iv) In how many ways can the letters of the words (a) CHEESES, (b) MISSISSIPPI be arranged?