## Kombinatorische Geometrie

SS 2000 Dr. Elsholtz

Aus Platons Timaios (http://classics.mit.edu/Plato/timaeus.1b.txt): übersetzt von Benjamin Jowett.

Im zweiten Absatz (Now is ...) werden die platonischen Körper konstruiert. Aus der Konstruktion ist (mehr oder weniger) offensichtlich, dass es keine weiteren geben kann. Die Tatsache, dass es genau fünf derartige Körper gibt, wird (noch viel ausführlicher) philosophisch gedeutet.

In the first place, then, as is evident to all, fire and earth and water and air are bodies. And every sort of body possesses solidity, and every solid must necessarily be contained in planes; and every plane rectilinear figure is composed of triangles; and all triangles are originally of two kinds, both of which are made up of one right and two acute angles; one of them has at either end of the base the half of a divided right angle, having equal sides, while in the other the right angle is divided into unequal parts, having unequal sides. These, then, proceeding by a combination of probability with demonstration, we assume to be the original elements of fire and the other bodies; but the principles which are prior to these God only knows, and he of men who is the friend God. And next we have to determine what are the four most beautiful bodies which are unlike one another, and of which some are capable of resolution into one another; for having discovered thus much, we shall know the true origin of earth and fire and of the proportionate and intermediate elements. And then we shall not be willing to allow that there are any distinct kinds of visible bodies fairer than these. Wherefore we must endeavour to construct the four forms of bodies which excel in beauty, and then we shall be able to say that we have sufficiently apprehended their nature. Now of the two triangles, the isosceles has one form only; the scalene or unequal-sided has an infinite number. Of the infinite forms we must select the most beautiful, if we are to proceed in due order, and any one who can point out a more beautiful form than ours for the construction of these bodies, shall carry off the palm, not as an enemy, but as a friend. Now, the one which we maintain to be the most beautiful of all the many triangles (and we need not speak of the others) is that of which the double forms a third triangle which is equilateral; the reason of this would be long to tell; he who disproves what we are saying, and shows that we are mistaken, may claim a friendly victory. Then let us choose two triangles, out of which fire and the other elements have been constructed, one isosceles, the other having the square of the longer side equal to three times the square of the lesser side.

Now is the time to explain what was before obscurely said: there was an error in imagining that all the four elements might be generated by and into one another; this, I say, was an erroneous supposition, for there are generated from the triangles which we have selected four kinds-three from the one which has the sides unequal; the fourth alone is framed out of the isosceles triangle. Hence they cannot all be resolved into one another, a great number of small bodies being combined into a few large ones, or the converse. But three of them can be thus resolved and compounded, for they all spring from one, and when the greater bodies are broken up, many small bodies will spring up out of them and take their own proper figures; or, again, when many small bodies are dissolved into their triangles, if they become one, they will form one large mass of another kind. So much for their passage into one another. I have now to speak of their several kinds, and show out of what combinations of numbers each of them was formed. The first will be the simplest and smallest construction, and its element is that triangle which has its hypotenuse twice the lesser side. When two such triangles are joined at the diagonal, and this is repeated three times, and the triangles rest their diagonals and shorter sides on the same point as a centre, a single equilateral triangle is formed out of six triangles; and four equilateral triangles, if put together, make out of every three plane angles one solid angle, being that which is nearest to the most obtuse of

plane angles; and out of the combination of these four angles arises the first solid form which distributes into equal and similar parts the whole circle in which it is inscribed. The second species of solid is formed out of the same triangles, which unite as eight equilateral triangles and form one solid angle out of four plane angles, and out of six such angles the second body is completed. And the third body is made up of 120 triangular elements, forming twelve solid angles, each of them included in five plane equilateral triangles, having altogether twenty bases, each of which is an equilateral triangle. The one element [that is, the triangle which has its hypotenuse twice the lesser side having generated these figures, generated no more: but the isosceles triangle produced the fourth elementary figure, which is compounded of four such triangles, joining their right angles in a centre, and forming one equilateral quadrangle. Six of these united form eight solid angles, each of which is made by the combination of three plane right angles; the figure of the body thus composed is a cube, having six plane quadrangular equilateral bases. There was yet a fifth combination which God used in the delineation of the universe.

Now, he who, duly reflecting on all this, enquires whether the worlds are to be regarded as indefinite or definite in number, will be of opinion that the notion of their indefiniteness is characteristic of a sadly indefinite and ignorant mind. He, however, who raises the question whether they are to be truly regarded as one or five, takes up a more reasonable position. Arguing from probabilities, I am of opinion that they are one; another, regarding the question from another point of view, will be of another mind. But, leaving this enquiry, let us proceed to distribute the elementary forms, which have now been created in idea, among the four elements.

To earth, then, let us assign the cubical

form; for earth is the most immoveable of the four and the most plastic of all bodies, and that which has the most stable bases must of necessity be of such a nature. Now, of the triangles which we assumed at first, that which has two equal sides is by nature more firmly based than that which has unequal sides; and of the compound figures which are formed out of either, the plane equilateral quadrangle has necessarily, a more stable basis than the equilateral triangle, both in the whole and in the parts. Wherefore, in assigning this figure to earth, we adhere to probability; and to water we assign that one of the remaining forms which is the least moveable; and the most moveable of them to fire; and to air that which is intermediate. Also we assign the smallest body to fire, and the greatest to water, and the intermediate in size to air; and, again, the acutest body to fire, and the next in acuteness to, air, and the third to water. Of all these elements, that which has the fewest bases must necessarily be the most moveable, for it must be the acutest and most penetrating in every way, and also the lightest as being composed of the smallest number of similar particles: and the second body has similar properties in a second degree, and the third body in the third degree. Let it be agreed, then, both according to strict reason and according to probability, that the pyramid is the solid which is the original element and seed of fire; and let us assign the element which was next in the order of generation to air, and the third to water. We must imagine all these to be so small that no single particle of any of the four kinds is seen by us on account of their smallness: but when many of them are collected together their aggregates are seen. And the ratios of their numbers, motions, and other properties, everywhere God, as far as necessity allowed or gave consent, has exactly perfected, and harmonised in due proportion.

Of all the writings of Plato the Timaeus is the most obscure and repulsive to the modern reader, and has nevertheless had the greatest influence over the ancient and mediaeval world...

And now I will explain to you the generation of the world by a method with which your scientific training will have made you familiar. Fire, air, earth, and water are bodies and therefore solids, and solids are contained in planes, and plane rectilinear figures are made up of triangles. Of triangles there are two kinds; one having the opposite sides equal (isosceles), the other with unequal sides (scalene). These we may fairly assume to be the original elements of fire and the other bodies; what principles are prior to these God only knows, and he of men whom God loves. Next, we must determine what are the four most beautiful figures which are unlike one another and yet sometimes capable of resolution into one another...Of the two kinds of triangles the equal-sided has but one form, the unequal-sided has an infinite variety of forms; and there is none more beautiful than that which forms the half of an equilateral triangle. Let us then choose two triangles; one, the isosceles, the other, that form of scalene which has the square of the longer side three times as great as the square of the lesser side; and affirm that, out of these, fire and the other elements have been constructed.

I was wrong in imagining that all the four elements could be generated into and out of one another. For as they are formed, three of them from the triangle which has the sides unequal, the fourth from the triangle which has equal sides, three can be resolved into one another, but the fourth cannot be resolved into them nor they into it. So much for their passage into one another: I must now speak of their construction. From the triangle of which the hypotenuse is twice the lesser side the three first regular solids are formed-first, the equilateral pyramid or tetrahedron; secondly, the octahedron; thirdly, the icosahedron; and from the isosceles triangle is formed the cube. And there is a fifth figure (which is made out of twelve pentagons), the dodecahedronthis God used as a model for the twelvefold division of the Zodiac.

Let us now assign the geometrical forms to their respective elements. The cube is the most stable of them because resting on a quadrangular plane surface, and composed of isosceles triangles. To the earth then, which is the most stable of bodies and the most easily modelled of them, may be assigned the form of a cube; and the remaining forms to the other elements.--to fire the pyramid, to air the octahedron, and to water the icosahedron,according to their degrees of lightness or heaviness or power, or want of power, of penetration. The single particles of any of the elements are not seen by reason of their smallness; they only become visible when collected. The ratios of their motions, numbers, and other properties, are ordered by the God, who harmonized them as far as necessity permitted.

The probable conclusion is as follows:-Earth, when dissolved by the more penetrating element of fire, whether acting immediately or through the medium of air or water, is decomposed but not transformed. Water, when divided by fire or air, becomes one part fire, and two parts air. A volume of air divided becomes two of fire. On the other hand, when condensed, two volumes of fire make a volume of air; and two and a half parts of air condense into one of water. Any element which is fastened upon by fire is cut by the sharpness of the triangles, until at length, coalescing with the fire, it is at rest; for similars are not affected by similars. When two kinds of bodies quarrel with one another, then the tendency to decomposition continues until the smaller either escapes to its kindred element or becomes one with its conqueror. And this tendency in bodies to condense or escape is a source of motion...Where there is motion there must be a mover, and where there is a mover there must be something to move. These cannot exist in what is uniform, and therefore motion is due to want of uniformity. But then why, when things are divided after their kinds, do they not cease from motion? The answer is, that the circular motion of all things compresses them, and as 'nature abhors a vacuum,' the finer and more subtle particles of the lighter elements, such as fire and air, are thrust into the interstices of the larger, each of them penetrating according to their rarity, and thus all the elements are on their way up and down everywhere and always into their own places. Hence there is a principle of inequality, and therefore of motion, in all time.

In the next place, we may observe that there are different kinds of fire–(1) flame, (2) light that burns not, (3) the red heat of the embers of fire. And there are varieties of air, as for example, the pure aether, the opaque mist, and other nameless forms. Water, again, is of two kinds, liquid and fusile. The liquid is composed of small and unequal particles, the fusile of large and uniform particles and is more solid, but nevertheless melts at the approach of fire, and then spreads upon the earth. When the substance cools, the fire passes into the air, which is displaced, and forces together and condenses the liquid mass. This process is called cooling and congealment. Of the fusile kinds the fairest and heaviest is gold; this is hardened by filtration through rock, and is of a bright yellow colour. A shoot of gold which is darker and denser than the rest is called adamant. Another kind is called copper, which is harder and yet lighter because the interstices are larger than in gold. There is mingled with it a fine and small portion of earth which comes out in the form of rust. These are a few of the conjectures which philosophy forms, when, leaving the eternal nature, she turns for innocent recreation to consider the truths of generation.

Water which is mingled with fire is called

liquid because it rolls upon the earth, and soft because its bases give way. This becomes more equable when separated from fire and air, and then congeals into hail or ice, or the looser forms of hoar frost or snow. There are other waters which are called juices and are distilled through plants. Of these we may mention, first, wine, which warms the soul as well as the body; secondly, oily substances, as for example, oil or pitch; thirdly, honey, which relaxes the contracted parts of the mouth and so produces sweetness; fourthly, vegetable acid, which is frothy and has a burning quality and dissolves the flesh. Of the kinds of earth, that which is filtered through water passes into stone; the water is broken up by the earth and escapes in the form of air-this in turn presses upon the mass of earth, and the earth, compressed into an indissoluble union with the remaining water, becomes rock. Rock, when it is made up of equal particles, is fair and transparent, but the reverse when of unequal. Earth is converted into pottery when the watery part is suddenly drawn away; or if moisture remains, the earth, when fused by fire, becomes, on cooling, a stone of a black colour. When the earth is finer and of a briny nature then two half-solid bodies are formed by separating the water,-soda and salt. The strong compounds of earth and water are not soluble by water, but only by fire. Earth itself, when not consolidated, is dissolved by water; when consolidated, by fire only. The cohesion of water, when strong, is dissolved by fire only; when weak, either by air or fire, the former entering the interstices, the latter penetrating even the triangles. Air when strongly condensed is indissoluble by any power which does not reach the triangles, and even when not strongly condensed is only resolved by fire. Compounds of earth and water are unaffected by water while the water occupies the interstices in them, but begin to liquefy when fire enters into the interstices of the water. They are of two kinds, some of them, like glass, having more earth, others, like wax, having more water in them.

Having considered objects of sense, we

now pass on to sensation. But we cannot explain sensation without explaining the nature of flesh and of the mortal soul; and as we cannot treat of both together, in order that we may proceed at once to the sensations we must assume the existence of body and soul.

What makes fire burn? The fineness of the sides, the sharpness of the angles, the smallness of the particles, the quickness of the motion. Moreover, the pyramid, which is the figure of fire, is more cutting than any other. The feeling of cold is produced by the larger particles of moisture outside the body trying to eject the smaller ones in the body which they compress. The struggle which arises between elements thus unnaturally brought together causes shivering. That is hard to which the flesh yields, and soft which yields to the flesh, and these two terms are also relative to one another. The yielding matter is that which has the slenderest base, whereas that which has a rectangular base is compact and repellent. Light and heavy are wrongly explained with reference to a lower and higher in place. For in the universe, which is a sphere, there is no opposition of above or below, and that which is to us above would be below to a man standing at the antipodes. The greater or less difficulty in detaching any element from its like is the real cause of heaviness or of lightness. If you draw the earth into the dissimilar air, the particles of earth cling to their native element, and you more easily detach a small portion than a large. There would be the same difficulty in moving any of the upper elements towards the lower. The smooth and the rough are severally produced by the union of evenness with compactness, and of hardness with inequality.