$$\mathbf{M} = \begin{pmatrix} \mathbf{2} & \mathbf{1} & \mathbf{-2} \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \\ \mathbf{-2} & \mathbf{2} & \mathbf{-1} \end{pmatrix}$$

```
{{2, 1, -2}, {1, 2, 2}, {-2, 2, -1}}
```

```
Eigenvalues[M]
```

```
{-3, 3, 3}
```

```
Eigenvectors[M]
```

```
{{1, -1, 2}, {-2, 0, 1}, {1, 1, 0}}
```

$$\mathbf{S = 1 / 6} \begin{pmatrix} \mathbf{Sqrt[6]} & \mathbf{-Sqrt[6]} & \mathbf{2\ Sqrt[6]} \\ \mathbf{3\ Sqrt[2]} & \mathbf{3\ Sqrt[2]} & \mathbf{0} \\ \mathbf{-2\ Sqrt[3]} & \mathbf{2\ Sqrt[3]} & \mathbf{2\ Sqrt[3]} \end{pmatrix}$$

$$\left\{\left\{\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}\right\},\right.$$

$$\left.\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}, \left\{-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}\right\}$$

```
Shochminus1 = Inverse[S]
```

$$\left\{\left\{\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}}\right\},\right.$$

$$\left.\left\{-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right\}, \left\{\sqrt{\frac{2}{3}}, 0, \frac{1}{\sqrt{3}}\right\}\right\}$$

S . M . Sminus1

$$\left\{\left\{-2+\frac{-\sqrt{\frac{2}{3}}-\frac{1}{\sqrt{6}}}{\sqrt{6}}-\frac{\sqrt{\frac{2}{3}}+\frac{1}{\sqrt{6}}}{\sqrt{6}},\right.\right.$$

$$\frac{-\sqrt{\frac{2}{3}}-\frac{1}{\sqrt{6}}}{\sqrt{2}}+\frac{\sqrt{\frac{2}{3}}+\frac{1}{\sqrt{6}}}{\sqrt{2}},$$

$$\left.-\sqrt{2}-\frac{-\sqrt{\frac{2}{3}}-\frac{1}{\sqrt{6}}}{\sqrt{3}}+\frac{\sqrt{\frac{2}{3}}+\frac{1}{\sqrt{6}}}{\sqrt{3}}\right\},$$

$$\left\{0,\sqrt{2}\left(\frac{1}{\sqrt{2}}+\sqrt{2}\right),0\right\},\{0,0,3\}\Big\}$$

Simplify[%]

```
{{-3, 0, 0}, {0, 3, 0}, {0, 0, 3}}
```