Exercise Sheet 1 Combinatorics
2014/15

Ex. 1
Let $\mathrm{ex}(n, C_3)$ denote the maximum number of edges a graph on n vertices can have, without containing a triangle (=cyclic graph of length 3) as a subgraph. Determine $\mathrm{ex}(n, C_3)$ as good as possible.

Ex. 2
Let $\mathrm{ex}(n, C_4)$ denote the maximum number of edges a graph on n vertices can have, without containing a cyclic graph of length 4.
Let p be prime, and let the set of vertices V be $V = \mathbb{Z}_p \times Z_p$. The vertices (x, y) and (x', y') are joined by an edge if $x + x' = yy'$. Show that (x, y) has at least $p - 1$ neighbours. From this, show that $|E| \geq \frac{1}{2}p^2(p-1)$. Show that there is no 4-cycle. (Suppose that (x, y) has two distinct neighbours $(x_1, y_1) \neq (x_2, y_2)$, (say). Show that, by the definition of the edges, y and then also x is uniquely defined.)
Would the same type of proof work if p is not a prime? (if yes, for which other values does it work?, if no, why not?)
Compare the new result on $\mathrm{ex}(n, C_4)$ with the previous ones.

Ex. 3
Given a set of n distinct points $p_1, \dots p_n \in P$ in the Euclidean plane, and a set of n distinct lines $l_1, \dots, l_n$. Prove that the number of point-line incidences (p_i, l_j) with $p_i \in l_j$ is $O(n^{3/2})$.
Hint: use the graph theoretic results we studied so far.

Ex. 4
Verify the number theoretic details (Lemma 1.8) of the proof of the Bruck-Ryser-Chowla theorem, see e.g. lecture notes by Simeon Ball, Zsuzsa Weiner, linked to on webpage.

Ex. 5 (Maybe with computer)
Give a list of 4 mutually orthogonal Latin squares of order 5.

Now let $p = 3$. Find an irreducible quadratic polynomial over $\mathbb{F}_3$. From this construct a finite field of 9 elements. (give its addition and multiplication table). From this, maybe with computer, give a list of 8 mutually orthogonal latin squares of order 9.

Ex. 6 (With computer)
Determine the number of Latin squares of order 6, which are essentially distinct. (Think about "normalizing" the Latin squares.) Try to prove: There are no two orthogonal Latin squares of order 6.

[Maybe you should start counting problems which are way too hard and tell me later the result...]
Please cross the problems you did in the Kreuze-system, link at webpage below. Deadline, Tuesday 21st October, 10am.

http://www.math.tugraz.at/~elsholtz/WWW/lectures/ws14/kombinatorik/vorlesung.html

Please note that by Nawi-regulatations, everybody who takes part in the exercise programme will receive a grade. (Deregistration from the Übungen is possible during October).