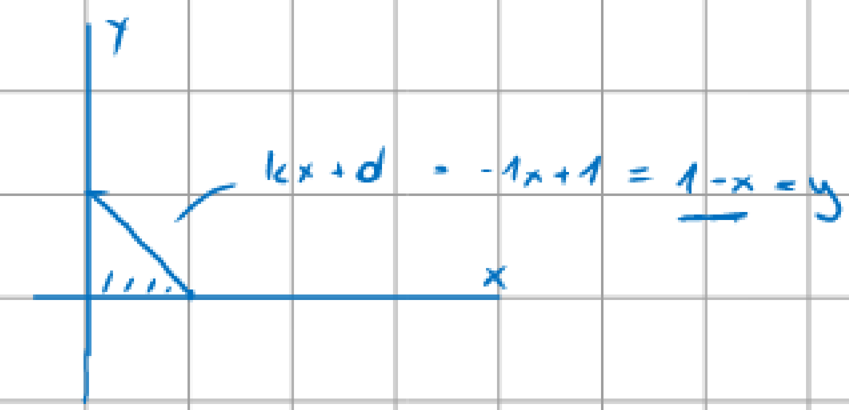
1. Berechnen Sie das Integral:
$$\iint_B x^2 y \, dx dy,$$
Dabei ist der Bereich B das
a) Dreieck mit den Ecken (0,0), (1,0), (0,1)
b) Dreieck mit den Ecken (0,1), (0,–1), (1,0)
c) Innere der Ellipse mit Halbachsen a, b in Hauptlage.

2. Berechnen Sie das Kurvenintegral
$$\oint_C (1 - x^2 + y^2) \, dx + (1 + x^2 - y^2) \, dy.$$
Dabei besteht C aus den Kurven
$$C_1 : y = \cos x, \ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$
$$C_2 : y = -\cos x, \ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$
und werde im Uhrzeigersinn durchlaufen.

3. Berechnen Sie das Kurvenintegral $\int_C (x^2 - y^2) \, dx + (x + y) \, dy$ längs der Berandung des Quadrats $|x| + |y| \leq 1$.

1) $\iint_B x^2 y \, dx \, dy$



a) $\triangle\ (0,0)\ (1,0)\ (0,1)$

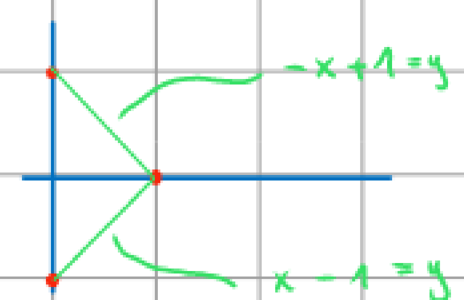
$$\int_{x=0}^{1}\left(\int_{y=0}^{1-x} x^2 y \, dy\right) dx = \int_{x=0}^{1}\left(\frac{x^2y^2}{2}\Bigg|_{y=0}^{1-x}\right) dx = \int_{x=0}^{1} \frac{x^2}{2}(1-x)^2 \, dx = \frac{1}{2}\cdot\left(\frac{x^5}{5} - \frac{\cancel{2}x^4}{\cancel{4}\,2} + \frac{x^3}{3}\Bigg|_0^1\right) =$$

$(x^2-2x+1)\,x^2\cdot\frac{1}{2}$

$(x^4-2x^3+x^2)\cdot\frac{1}{2}$

$$= \frac{1}{2}\cdot\left(\frac{1}{5}-\frac{1}{2}+\frac{1}{3}\right) = \underline{\frac{1}{60}}$$

b) $\triangle\ (0,1)\ (0,-1)\ (1,0)$



$$\int_{x=0}^{1}\left(\int_{y=x-1}^{1-x} x^2 y \, dy\right) dx$$

$$\int_{x=0}^{1}\left(\frac{x^2y^2}{2}\Bigg|_{y=x-1}^{1-x}\right) dx = \int_{x=0}^{1} 0 = \underline{0}$$

$$\frac{x^2(1-x)^2}{2} - \frac{x^2(x-1)^2}{2}$$

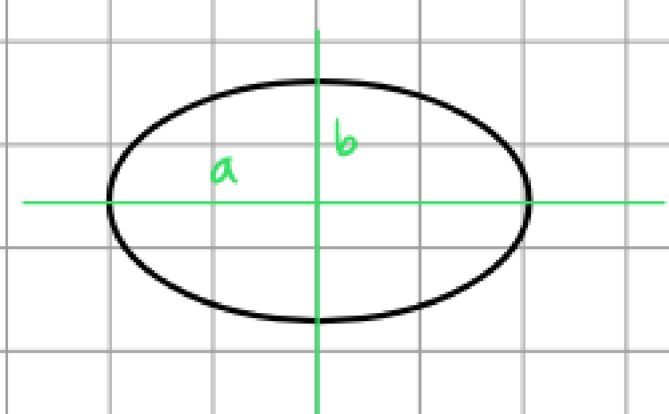
$$\frac{x^2}{2}\left((1-x)^2-(x-1)^2\right)$$

$$\left(x^2-2x+1-(x^2-2x+1)\right)$$

$= 0$

c) Ellipse Halbachsen a, b

$\iint_B x^2 y \, dx \, dy$



$$x^2/a^2 + y^2/b^2 = 1$$

$$\frac{x^2}{a^2} - 1 = -\frac{y^2}{b^2} \quad \Big| \cdot (-b^2)$$

$$-\frac{x^2\cdot b^2}{a^2} + b^2 = y^2 \quad \Big| \sqrt{}$$

$$\sqrt{b - \frac{x^2b^2}{a^2}} = y$$

$$= \int_{x=-a}^{a}\left(\int_{y=-\sqrt{b-\frac{x^2b^2}{a^2}}}^{\sqrt{b-\frac{x^2b^2}{a^2}}} x^2 y \, dy\right) dx$$

$$= \int_{x=-a}^{a}\left(\frac{x^2y^2}{2}\Bigg|_{y=-\sqrt{b-\frac{x^2b^2}{a^2}}}^{\sqrt{b-\frac{x^2b^2}{a^2}}}\right) dx = \int_{x=-a}^{a} \frac{x^2}{2}\cdot\left(b-\frac{x^2b^2}{a^2}\right) - \frac{x^2}{2}\left(b-\frac{x^2b^2}{a^2}\right) \, dx =$$

$$= \int_{x=-a}^{a} 0 \, dx = \underline{0}$$

2. $\oint (1-x^2+y^2)\,dx + (1+x^2-y^2)\,dy$

C_1: $y = \cos x \,, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$\vec{x} = \begin{pmatrix} t \\ \cos t \end{pmatrix} \qquad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \qquad \dot{\vec{x}} = \begin{pmatrix} 1 \\ -\sin(t) \end{pmatrix}$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1-t^2+\cos^2(t)\right)\cdot 1 + \left(1+t^2-\cos^2(t)\right)\cdot(-\sin(t))\,dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1-t^2+\cos^2(t) - \sin t + t^2 \sin t + \cos^2(t)\sin t \; dt$$

$$\left[t - \frac{t^3}{3} + \frac{t+\sin(t)\cos(t)}{2} + \underset{0}{\underset{\shortparallel}{\cos(t)}} + \underset{0}{\underset{\shortparallel}{2t\sin(t)}} - \underset{0}{\underset{\shortparallel}{(t^2-2)\cdot\cos(t)}} - \underset{0}{\underset{\shortparallel}{\frac{1}{3}\cos^3(t)}} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\pi - \frac{\pi^3}{12} + \frac{\pi}{2} = \frac{3\pi}{2} - \frac{\pi^3}{12}$$

C_2: $y = -\cos(x) \,, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \qquad \vec{x} = \begin{pmatrix} t \\ -\cos t \end{pmatrix} \qquad \dot{\vec{x}} = \begin{pmatrix} 1 \\ \sin t \end{pmatrix}$

$\frac{\pi}{2} \leq t \leq -\frac{\pi}{2}$

$$\int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} \left(1-t^2+\cos^2(t)\right) + \left(1+t^2-\cos^2(t)\right)\cdot \sin(t)\,dt$$

$$\int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} \left(1-t^2+\cos^2(t)\right) + \sin(t) + t^2\cdot\sin(t) - \cos^2(t)\cdot\sin(t) \quad dt$$

$$= -\pi + \frac{\pi^3}{3} - \frac{\pi}{2} + 0 = -\frac{3\pi}{2} + \frac{\pi^3}{12}$$

$$C_1 + C_2 = \cancel{\frac{3\pi}{2}} - \cancel{\frac{\pi^3}{12}} - \cancel{\frac{3\pi}{2}} + \cancel{\frac{\pi^3}{12}} = 0$$

3)

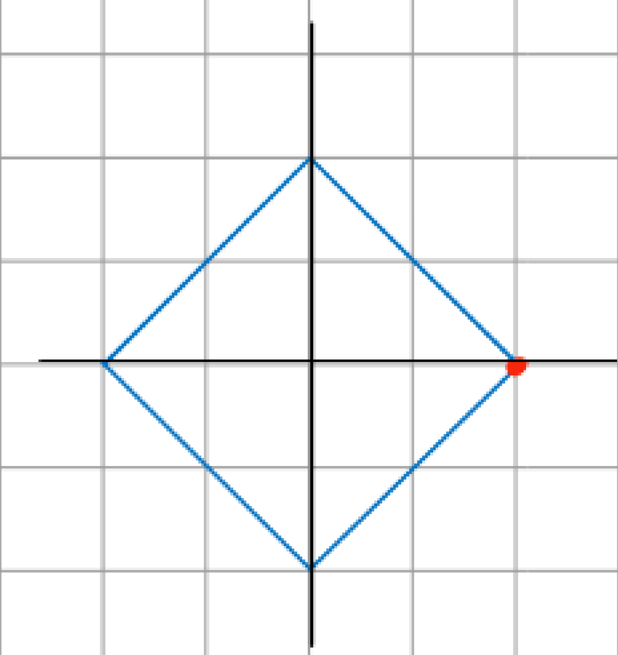
$$\int_C (x^2-y^2)\,dx + (x+y)\,dy \qquad |x|+|y| \leq 1$$

Potenzialfunktion

$$\frac{\partial P}{\partial y} = -2y$$

$$\frac{\partial Q}{\partial x} = 1$$

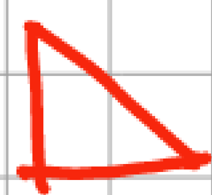
$\neq$



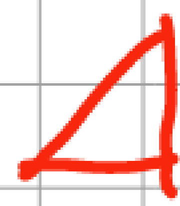
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-t \\ t \end{pmatrix} \qquad \begin{matrix} dx \to \\ dy \to \end{matrix} \dot{x}(t) = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \begin{matrix} dt \\ dt \end{matrix}$$

$t \in [0 \dots 1]$

$$\int_C \langle v, \dot{x}(t) \rangle \, dt$$

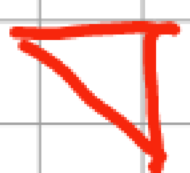


$$\int_{t=0}^{1} -\big(\underbrace{(1-t)^2 - t^2}_{\cancel{t^2}-2t+1-\cancel{t^2}}\big) + (1\cancel{-t}\cancel{+t})\, dt = \int_{t=0}^{1} 2t = t^2 \Big|_0^1 = \underline{1}$$



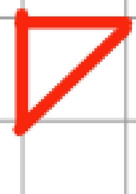
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -t \\ 1-t \end{pmatrix} \qquad \dot{x}(t) = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \qquad t \in [0 \dots 1]$$

$$\int_{t=0}^{1} \underbrace{\big(t^2 - (1-t^2)\big)}_{2t-1} - \underbrace{\big(-t + (1-t)\big)}_{+2t-1}\, dt = \int_{t=0}^{1} 4t-2 \, dt = 2t^2-2t \Big|_0^1 = \underline{0}$$



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t-1 \\ -t \end{pmatrix} \qquad \dot{x}(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad t \in [0 \dots 1]$$

$$\int_{t=0}^{1} \underbrace{\big((t-1)^2 - t^2\big)}_{\substack{t^2-2t+1-t^2 \\ -2t+1}} - \underbrace{\big(-t + (t-1)\big)}_{+1}\, dt = \int_{t=0}^{1} -2t+2 \, dt = -t^2+2t \Big|_0^1 = \underline{1}$$



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ t-1 \end{pmatrix} \qquad \dot{x}(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad t \in [0 \dots 1]$$

$$\int_{t=0}^{1} \underbrace{\big(t^2 - \underbrace{(t-1)^2}_{t^2-2t+1}\big)}_{2t-1} + \underbrace{\big(t + (t-1)\big)}_{2t-1}\, dt = \int_{t=0}^{1} 4t-2 \, dt = 2t^2-2t \Big|_0^1 = \underline{0}$$

Summe $= 1 + 0 + 1 + 0 = \underline{\underline{2}}$