

# Themen Bakkalaureatsprojekt TM

## Finanz- und Versicherungsmathematik

### 2012

#### 1. Calibration of the Ornstein-Uhlenbeck process using both least squares regression and maximum likelihood estimation (ecofinance)

In mathematics, the Ornstein-Uhlenbeck process, is a stochastic process that is stationary, Gaussian, and Markovian, and is the only nontrivial process that satisfies these three conditions, up to allowing linear transformations of the space and time variables. Over time, the process tends to drift towards its long-term mean: such a process is called mean-reverting.

The process can be considered to be a modification of the random walk in continuous time, or Wiener process, in which the properties of the process have been changed so that there is a tendency of the walk to move back towards a central location, with a greater attraction when the process is further away from the centre. The Ornstein-Uhlenbeck process can also be considered as the continuous-time analogue of the discrete-time AR(1) process.

An Ornstein-Uhlenbeck process,  $x_t$ , satisfies the following stochastic differential equation:

$$dx_t = \theta(\mu - x_t)dt + \sigma dW_t$$

where  $\theta > 0$ ,  $\mu, \sigma > 0$  are parameters and  $W_t$  denotes the Wiener process.

The Ornstein-Uhlenbeck process is one of several approaches used to model (with modifications) interest rates, currency exchange rates, and commodity prices stochastically. The parameter  $\mu$  represents the equilibrium or mean value supported by fundamentals;  $\sigma$  the degree of volatility around it caused by shocks, and  $\theta$  the rate by which these shocks dissipate and the variable reverts towards the mean.

To simulate interest rates, currency exchange rates of commodity prices with the Ornstein-Uhlenbeck process, the parameters of the process must be estimated using historical data. Two common methods of parameter estimation are the Least Squares Regression method and the Maximum Likelihood Estimation.

#### 2. Comparison of the Ornstein-Uhlenbeck process to non-mean-reversion processes (like Ho-Lee model, geometric Brownian motion, Ornstein-Uhlenbeck process without the mean-reversion term) (ecofinance)

In mathematics, the Ornstein-Uhlenbeck process, is a stochastic process that is stationary, Gaussian, and Markovian, and is the only nontrivial process that satisfies these three conditions, up to allowing linear transformations of the space and time variables. Over time, the process tends to drift towards its long-term mean: such a process is called mean-reverting.

An Ornstein-Uhlenbeck process,  $x_t$ , satisfies the following stochastic differential equation:

$$dx_t = \theta(\mu - x_t)dt + \sigma dW_t$$

where  $\mu$  represents the equilibrium or mean value supported by fundamentals,  $\sigma > 0$  is the degree of volatility around it caused by shocks,  $\theta > 0$  is the rate by which these shocks dissipate and the variable reverts towards the mean and  $W_t$  denotes the Wiener process.

This process is very common under practitioners. It shall be compared with non-mean-reverting processes such like the Ho-Lee model, the geometric Brownian motion or the Ornstein-Uhlenbeck process without the mean-reversion term. Then the pros and cons of each process shall be discussed.

### 3. **Principal component analysis for interest rate curves, commodity curves or FX curves (ecofinance)**

Principal component analysis (PCA) is a mathematical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. The number of principal components is less than or equal to the number of original variables. This transformation is defined in such a way that the first principal component has the largest possible variance (that is, accounts for as much of the variability in the data as possible), and each succeeding component in turn has the highest variance possible under the constraint that it be orthogonal to (i.e., uncorrelated with) the preceding components. Principal components are guaranteed to be independent only if the data set is jointly normally distributed. PCA is sensitive to the relative scaling of the original variables.

Because interest rate, FX and commodity curves consist of many components (often up to 30 points) but are correlated with each other, PCA is used to reduce the amount of data that needs to be simulated.

For modeling the curves, the Ornstein-Uhlenbeck process shall be used.

### 4. **Comparison of 1-factor with multi-factor models (ecofinance)**

A financial model that employs multiple factors in its computations to explain market phenomena and/or equilibrium asset prices is called a multi-factor model. The multi-factor model can be used to explain either an individual security or a portfolio of securities. It will do this by comparing two or more factors to analyze relationships between variables and the resulting performance of the securities.

In practice, interest rates, foreign exchange rates or commodity prices are modeled using several different approaches. There are "simple" ones, where the stochastic process is modeled using only one factor (like the Vasicek or Hull-White model), and more "advanced" ones where more factors are used to model the market data.

The aim of this work is to discuss the pros and cons of several one- and multi-factor models and to compare their performance when applied to real life market data.

1-factor models:

- Vasicek
- Hull-White
- Ho-Lee
- Black-Karasinski
- Black-Derman-Toy
- Cox-Ingersoll-Ross

Multi-factor models:

- 2-factor Hull-White
- Longstaff-Schawrtz
- Chen

### 5. **Marktanalyse und Konzipierung von Microversicherungen bei Großevents (Liland)**

Ziel dieser Arbeit ist es einerseits durch Umfragen und Marktanalyse einen möglichen Markt für Mikroversicherungen im Evenbereich zu identifizieren. Zusätzlich sollen Konzepte und Preisgestaltungen für neue Versicherungsprodukte entwickelt werden. Beispiele solcher Versicherungen sind kurzzeitiger Krankenversicherungen die nur für die Dauer des Besuches einer Großveranstaltung (z.B. Konzert) gelten.

**6. Erstellung eines Lebenserwartungsrechners für Risikopatienten (LMS)**

Ziel dieser Arbeit ist es auf Grund von medizinischen Daten eine spezielle Sterbetafel für Patienten mit Herzerkrankungen zu erstellen. Ein dazugehöriges Berechnungsprogramm soll in Microsoft Excel umgesetzt werden.

**7. Anwendung des Rückversicherungsprogrammes in den Solvency II-NatCat Berechnungen: Angemessene Berücksichtigung des Summenexzedenten (Grawe)**

Problemstellung: In den SII-Berechnungen zu den Naturgefahren wird entsprechend dem Standardmodell der Bruttoschaden eines Versicherungsunternehmens nur gesamt ermittelt. Für die Berechnung des Nettoschadens bei Vorliegen eines Summenexzedenten ist es jedoch erforderlich die Höhe des Schadens auf Einzelvertragsbasis zu ermitteln.

Ziel: Eigenständige Entwicklung alternativer Lösungswege

Zur Verfügung gestellt werden:

- Excel mit Ausschnitt aus den SII-Standardmodell Berechnungen zur Naturgefahr Hagel
- Beispiel eines Rückversicherungsprogrammes bestehend aus: Summenexzedent, Quotenrückversicherung und Stop Loss
- Sampledaten mit Versicherungssummen je CRESTA-Zone (Postleitzahlen 2Steller)