Mathematical Analysis of Algorithms.

Exercises V. (25.06.2012)

1. a) (0.5pt) Show that the Catalan numbers admit the following expansion:

$$\frac{1}{n+1}\binom{2n}{n} = \frac{4^n}{n\sqrt{\pi n}}\left(1 - \frac{9}{8n} + \frac{145}{128n^2} + O\left(\frac{1}{n^3}\right)\right)$$

   b) (0.5pt) Derive an asymptotic expansion for

$$\sum_{1\le k\le n} \frac{1}{k^2}$$

   to within $O(\frac{1}{n^3})$.

   c) (0.5pt) Derive an asymptotic expansion for

$$\sum_{1\le k\le n} \frac{1}{\sqrt{k}}$$

   to within $O(\frac{1}{n^2})$.

2. Consider the function

$$f(z) = \frac{1}{(1-z^3)^2(1-z^2)^3(1-\frac{z^2}{2})}.$$

   - (1pt) Derive an asymptotic estimate of $[z^n]f(z)$.

3. A graph is called *outerplanar* if it can be embedded in the plane in such a way that every vertex lies on one common face. Let $\mathcal{B}$ be the class of all 2-connected labelled outerplanar graphs and $\mathcal{B}_n$ the set of all graphs in $\mathcal{B}$ on exactly n vertices.

   - (1pt) Derive the asymptotic estimate of $b_n := |\mathcal{B}_n|$.

4. Let $\mathcal{P}$ be the class of all permutations with cycles of *even* length and $\mathcal{P}_n$ be the set of permutations in $\mathcal{P}$ on $[n]$.

   - (1pt) Find the asymptotic expansion of $p_n := |\mathcal{P}_n|$ up to the third order term.