GRAZ UNIVERSITY OF TECHNOLOGY
INSTITUTE OF ANALYSIS AND NUMBER THEORY
Marc Technau

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2. exercise sheet for Mathematics for advanced materials science

2.1. *(Linear ordinary differential equations)*

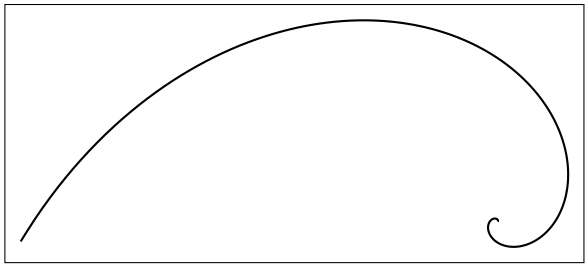
(a) Let λ_0 be an arbitrary number. Verify that both $t \mapsto \exp(\lambda_0 t)$ and $t \mapsto t\exp(\lambda_0 t)$ satisfy the differential equation $\ddot{x} - 2\lambda_0\dot{x} + \lambda_0^2 x \stackrel{!}{=} 0$.

(b) Find a solution x to the differential equation from (a) with $x(0) = 1$ and $\dot{x}(0) = 0$.

2.2. *(Powers of complex numbers)*
For $n \in \mathbb{N}$ let $z_n = (1 + \mathrm{i}\sqrt{3})^n$.

(a) Write z_n in polar form and sketch the three points z_1, z_2 and z_3 in the complex plane.

(b) Find a formula for the real and imaginary parts of z_n for $n \in \mathbb{N}$. (Hint: use the polar form of z_1 to compute z_n.)



2.3. *(Complex differentiation)*
Let z be a complex number. Compute:

(a) $\dfrac{\mathrm{d}}{\mathrm{d}z}\left(z^5 + 4z^2 - \sin(z) + 42\right)$,

(b) $\dfrac{\mathrm{d}}{\mathrm{d}z}\left(\exp\left(\dfrac{z^2}{z+1}\right)(1+z)^2\right)$ for $z \neq -1$.

Note: this exercise sheet (like any other with an even number) will be discussed during class. Please make sure that you have prepared enough notes to explain how you have arrived at your solution. Moreover, all of your notes should be uploaded to the TeachCenter.

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 19.10.2021, 23:55 o'clock. `https://tc.tugraz.at/main/course/view.php?id=3543`
`https://www.math.tugraz.at/~mtechnau/teaching/2021-w-mams.html`